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THE
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VOLUME XXXIX MAY 1914 NUMBER 4

A DETERMINATION OF THE SUN'S TEMPERATURE

By GLENN A. SHOOK

INTRODUCTION

In 1906 Moissan carried out a number of experiments upon the vaporization of metals.[1] He placed the temperature of his furnace at 3500° C. and made the statement that all known elements volatilize at that temperature. Now it is thought by Schulz that the temperature of the furnace must have been considerably above 3500° C. and probably as high as the sun's photosphere which he sets at 5400° C.[2] He argued that owing to the large current used by Moissan there was an enormous amount of energy which had no adequate escape by conduction or radiation and which therefore must have raised the temperature of the furnace up to the point where it was checked by the melting and evaporization of the limestone of which it was constructed. He moreover asserts that the volatilization of the metals is not to be regarded as complete.

We also find the following remarks in regard to molybdenum and tungsten:

Molybdenum.—The 150 grams were not fused by a current of 500 amperes and 110 volts. After applying 700 amperes and 110 volts for seven minutes,

[1] *Annales de chemie et de physique*, **8**, 151, 1906.

[2] *Astrophysical Journal*, **29**, 33, 1909.

the metal was fused but nothing evaporated. After twenty minutes 56 grams were distilled.

Tungsten.—After applying 500 amperes and 110 volts for 5 minutes the metal was not yet fused. After applying 800 amperes and 110 volts for twenty minutes, boiling commenced but only 25 grams distilled.

It thus appears that the volatilization is partly a question of time, and when we remember that the sun's photosphere is probably at a temperature of 8000° C. or 9000° C. and that such a temperature has existed for years and not minutes, we must conclude that all elements in the sun are necessarily in the gaseous state.

The following hypothesis which has been advanced by a number of investigators[1] is confirmed by the present research.

In the first place the material of the sun is "gaseous," that is, it follows the extended law for gases.

Secondly, the radiation that reaches us comes from the reversing layer alone or at least only from the superficial layers of the photosphere.

Thirdly, there is a relatively large drop in the temperature at the reversing layer.

If there is considerable scattering of light due to the gases of the reversing layer, then the light that reaches us comes from a small depth only. Moreover, the scattering is greater for blue light than for red, consequently the blue part of the spectrum must be relatively weaker than the red part. Hence, if the temperature falls off rapidly as we move outward radially through the reversing layer we should expect the temperature for blue light to be less than that determined for red light. Also as we move across the sun's disk, we should expect the apparent temperature to fall off rapidly as we approach the limb and we should, moreover, expect the temperature gradient for blue to be greater than that for red. This is precisely what the writer finds. The sharp boundary of the photosphere is additional proof of the gaseous scattering.

That the scattering prevents us from seeing beyond a shallow depth of the reversing layer may be shown by a rough calculation.

[1] Secchi, *Le soleil*, **1**, Book III, chap. iv, p. 267; **2**, Book VII, chap i, p. 299; Schwarzschild, "Ueber das Gleichgewicht der Sonnenatmosphäre," *Göttingen Nachr., Math. Phys. Kl.*, 1906, pp. 1–13; Abbot, *The Sun*, p. 236.

The law of molecular absorption is expressed by the following formula:

$$I=I_0e^{-kh}$$

or

$$\log\frac{I}{I_0}=-kh$$

where $I_0=$ the intensity of light incident upon the absorbing medium; $I=$the intensity of the transmitted light; $k=$the fraction of light absorbed by unit thickness of the medium; and $h=$the thickness or height of the absorbing layer.

Using Abbot's[1] values for the transmission of the atmosphere above Mount Wilson we have:

Wave-length in μ.	0.4	0.5	0.6	0.7
Percentage of transmission. . . .	76	89	95	97

Taking the Mount Wilson atmosphere, which is about 10 miles, as our unit thickness, the length of a column of gas for an extinction of 99 per cent or a transmission of 1 per cent for a wave-length of 0.4 μ becomes

$$\frac{\log 0.01}{0.4343}=-0.24h$$

or

$$h=18.5\ ,$$

that is, the column would have to be 185 miles if the gas had the same density as the Mount Wilson atmosphere.

The relative densities of the photosphere and the Mount Wilson atmosphere may be determined by means of Boyle's Law as follows:

Let the pressure, volume, and absolute temperature of the former be p', v', and T', and the corresponding quantities for the latter be p, v, and T. We may now write:

$$pv=RT$$

and

$$p'v'=RT'$$

hence

$$\frac{pv}{p'v'}=\frac{T}{T'}\cdot$$

[1] *Nature*, **81**, 97, 1909.

Assuming that the pressure of the reversing layer is about 5 atmospheres, that its mean temperature is 7000° A., and that the temperature of the earth's atmosphere is 250° A., we obtain the relation:

$$\frac{1\times v}{5\times v'}=\frac{250}{7000}.$$

Writing d_s for the density of the reversing layer and d_e for the density of the earth's atmosphere, the above equation becomes:

$$\frac{d_s}{d_e}=\frac{250\times 5}{7000}.$$

Hence a column of gas on the sun sufficient to produce an extinction of 99 per cent at wave-length 0.4 μ would have to be

$$18.5\times 10\times\frac{7000}{250\times 5}=1000 \text{ miles high}.$$

In this manner Table I was constructed

TABLE I

Wave-Length	Miles
0.4 μ	1000
0.5	2400
0.6	5200
0.7	8600

Since the radius of the sun is 435,000 miles, it is readily seen that the radiation which we are utilizing for the estimation of temperature comes from only the outermost solar layers. It is also observed that for short wave-lengths the depth to which we are able to penetrate is smaller than that which obtains for the longer wave-lengths.

EXPERIMENTAL METHOD FOR THE DIRECT DETERMINATION OF THE SUN'S APPARENT TEMPERATURE

The method employed by the writer for the determination of the sun's apparent temperature is an application of Planck's formula for the visible spectrum. In this method the brightness of the sun's disk is compared photometrically with the brightness of the filament of a miniature incandescent lamp for three different colors. To carry out these observations the Department of

Astronomy of this university kindly permitted the use of their small observatory, which is equipped with a six-inch equatorial telescope. A new eyepiece was constructed, providing a receptacle for the lamp between the eye-lens and the field-lens. A new finder, provided with a micrometer scale and parallel hairs, was also attached to the telescope.

An image of the sun is formed by the objective in the focal plane of the eyepiece. The incandescent lamp is adjusted until its filament lies in the plane of the image of the sun's disk.

If one looks through the telescope when it is directed toward the sun he sees the image of the lamp-filament superimposed upon the image of the sun's disk. Now by varying the current through the lamp the filament can be made to disappear against the bright background of the sun's image. When this condition obtains, the temperature of the filament is equal to the apparent black-body temperature of the image, and by means of Planck's formula the apparent black-body temperature of the sun's disk can be estimated if the temperature of the filament is known as a function of the current through the lamp.

In the present investigation the lamps used were calibrated by the Bureau of Standards. The eyepiece that was used in the equatorial and which contained a lamp receptacle was fitted into a small telescope and this arrangement was used by the bureau in calibrating the lamps by means of their standard black body. They furnished for each lamp a table containing a series of temperatures and the corresponding currents through the lamp. The error which might be caused by reflections from the lamp globe and eyepiece lenses was thus eliminated.

As a matter of fact, the entire filament will never disappear since all parts are not of the same intensity, but one always uses the central portion of the tip and this is practically uniform in intensity.

The filament (Fig. 2) may be moved about easily to any point on the disk, which is represented by the dotted line, by means of the right ascension and the declination screws. Fig. 1 shows the reticle of the finder with the scale and parallel spider lines. These parallel lines are adjusted so that their distance apart is equal to

the diameter of the sun's image, and they are, moreover, always parallel to the ecliptic.

The axis of the lamp is generally maintained perpendicular to the ecliptic. The lamp is connected in series to a few storage cells, an adjustable resistance, and a milliammeter (Fig. 3).

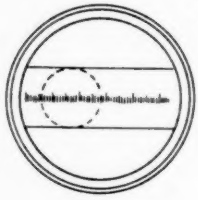

FIG. 1

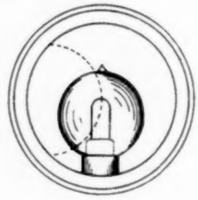

FIG. 2

This arrangement of lamp and eyepiece, which is the result of some experimenting, was found to be the most satisfactory. With an equatorial as small as this one the image is only about 1 cm in diameter, and in order to investigate the intensity along any radius, i.e., along a distance of 0.5 cm, with any accuracy it is necessary

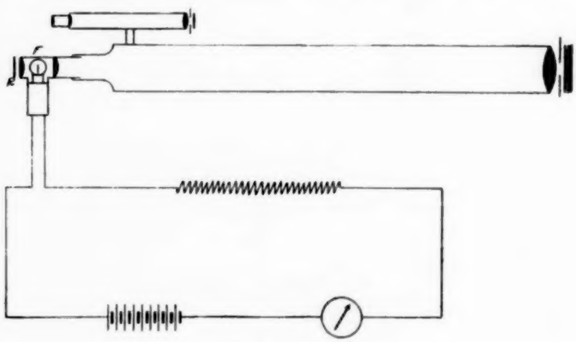

FIG. 3

to have a rather large magnification. In order to obtain a clear image the field-lens is also indispensable. Again, with the present arrangement the globe of the lamp just about fills up the space between the two lenses and therefore it is not in focus; consequently when one looks at the tip of the filament the contour of the globe is scarcely noticed. If an eye-lens of longer focal length were used, the globe would cause a distortion of the image.

The problem of diminishing the intensity of the sun's image to that of an incandescent lamp-filament presents no small difficulty. The intensity may be partly diminished by diaphragming down the objective, but one cannot resort solely to this method without seriously impairing the definition of the image. When the aperture is made as small as is permissible, an absorption glass may be used, but it is almost necessary to use three or more if the absorption coefficient of the arrangement is required in any calculation. The density of a single glass required to make the necessary reduction in intensity is so great that it is impossible to measure its absorption coefficient with any accuracy. Since the absorption of these glasses is never absolutely general, i.e., non-selective, and since they differ slightly among themselves, it is necessary to measure the absorption factor of each glass for each wavelength used.

Moreover, the optical properties of these glasses must be almost as good as those of the telescope objective; otherwise aberrations would result. It is for this reason that it is practically impossible to use a large telescope since the absorption glasses would have to be made with as much care as the objective of the telescope.

In order to determine the best arrangement for the six-inch equatorial used in this investigation a number of observations were carried out upon the moon's disk. The most conspicuous craters were carefully studied with a full objective and then with a number of diaphragms having apertures of different size. In this manner it was found that an aperture of about 1.5 cm still produced good definition. In addition to this diminishing of the aperture, three absorption glasses were also used. The objective of the finder was also stopped down and in addition an absorption glass was used.

Monochromatic light was produced by placing colored glasses directly before the eye-lens *R* (Fig. 3). It is practically impossible to obtain a single colored glass which is even approximately monochromatic. Four colors of Jena glass were obtained from Petitdidier, Chicago—namely, red, yellow, green, and blue. The red is remarkably good, transmitting only a red band, and that rather narrow. The yellow, which appeared monochromatic to the

unaided eye, was found to transmit almost the entire spectrum. The green contains a faint band in the yellow but it is free from blue. The blue glass transmits a band in the red, as is usually the case with blue glasses, and also faint lines in the green.

While, according to our information, these are the best glasses that can be obtained, it is readily seen that they were unsuitable without some modifications.

A detailed study of monochromatism of various kinds of glass was then undertaken. A quantity of different kinds of colored glass was obtained and these glasses were all examined separately by means of a spectroscope, and then different combinations were tried until the best arrangement was obtained. The Jena glasses were found to be superior to any examined but a combination of three different glasses was found to give the best results.

For example, some green glass transmits blue light but no red, while nearly all blue glass transmits some red; consequently a combination of the two is practically free from red without any perceptible reduction of the blue light.

In this manner it was possible to obtain combinations for red, green, and blue light, all of which are practically monochromatic. The search for monochromatic yellow was, however, futile. It seems almost impossible to obtain a glass or a combination of glasses which produces yellow and excludes all the other colors in even a moderate degree.

The fact that a glass for a particular color may contain a faint band of another color is often of no consequence, providing that consistent readings may be made, and a very narrow band is not always necessary if the band contains only one color. For instance, we may have a rather wide red band, but so long as there is no orange included in the band a good photometric balance can always be made, and the wave-length used would always be the central part of the band. The difference between this wave-length and the true optical center of gravity is too insignificant to consider in this particular problem.

There are other methods for producing monochromatic light, but none is very well suited to this particular problem. The spec-

troscopic eyepiece designed by Mendenhall[1] for pyrometers using the disappearing-filament principle is best adapted to this particular apparatus, but it was rejected for several reasons. In Mendenhall's pyrometer a short horizontal section of the lamp-filament and the superimposed image are focused upon the slit of an auxiliary direct-vision spectroscope. The slit of the spectroscope is vertical so that the field is crossed by three spectra, the middle one corresponding to the lamp-filament. A diaphragm is so placed in the focal plane of the eyepiece of the spectroscope that only the desired region of the spectrum is transmitted to the eye. In order that this central band may be wide enough to make a photometric comparison it is necessary to use a very thick lamp-filament, and this is impossible when a large magnification of the image is required as in the present investigation, for then all parts of the filament would not be in focus. Even with a fine filament there is some distortion of the image.

Furthermore, any such spectroscopic method diminishes the intensity of the light considerably, making it necessary in the blue and violet region to open both slits of the instrument very wide in order to get sufficient light to make a balance. If this is done, we have no longer strictly monochromatic light, and we may as well employ colored glasses. With a colored glass one sees the filament and sun's image directly so that he always knows just what part of the disk he is on, but with the spectroscopic eyepiece this of course is not the case, and he must depend entirely upon the finder.

It has been shown by a number of experimenters that the disappearing-filament principle is by far the most sensitive photometric scheme that we have, and it is particularly adapted to this problem, since one can move the filament about to any point on the sun's image, and make a temperature measurement at that point.

The wave-lengths of the monochromatic glasses were determined by means of a Lummer-Brodhun spectrophotometer made by Schmidt and Hensch. The same instrument was also used to determine the absorption coefficients of the absorption glasses.

[1] *Physical Review*, **33**, 1, 1911.

DATA AND RESULTS

1. *Wave-lengths of the monochromatic glasses.*—The readings of the arbitrary scale of the Lummer-Brodhun spectrophotometer for the three colored glasses used are given in the following tables:

TABLE II

OCULAR SLIT=0.05 CM SLIT No. 1, 50

Red Glass		Green Glass		Blue Glass	
Blue End	Red End	Blue End	Red End	Blue End	Red End
616	546	734	654	994	750
618	546	732	654	994	748
614	544	734	652	990	750
618	546	736	654	994	748
616	542	736	652	994	752
616	546	734	656	1,000	750
616	544	734	656	1,002	748
616	546	736	654	996	750
614	546	736	654	994	748
614	544	734	654	996	754
Mean..........580		Mean..........694		Mean..........868	

The blue light was very faint, hence the readings are not quite so consistent as in the case of the red and green.

The wave-lengths in μ, corresponding to these arbitrary scale readings, are as follows:

TABLE III

	Lummer-Brodhun Scale Reading	Wave-Length in μ
Red glass...........	580	0.661
Green glass..........	694	0.537
Blue glass...........	868	0.446

2. *Absorption factors.*—In determining the absorption factor R for any particular glass the zero reading of the Lummer-Brodhun spectrophotometer was taken before and after the observations with the glass.

The standard lamps were connected in parallel to the same mains and the voltage was controlled by a rheostat.

TABLE IV

Red Light — L.-B. No. 580
Ocular Slit = 0.05 cm

Zero Reading	Volts	Glass No. 1	Volts
Slit No. 1, 50.0		Slit No. 1, 100	
Slit No. 2, 51.0	107.0	Slit No. 2, 8.4	107.0
51.9		8.3	
50.6		8.4	
50.9	107.0	8.3	107.0
51.0		8.3	
51.9		8.4	
51.0		8.4	107.0
51.4		8.4	
52.0		8.3	
51.3	107.0	8.3	107.0
Mean . . 51.30		Mean 8.35	

Glass No. 1	Volts	Glass No. 1	Volts
8.3	107.0	8.3	107.0
8.5		8.5	
8.4		8.3	
8.4		8.4	
8.3		8.3	
8.5		8.2	
8.5		8.3	
8.5	107.0	8.3	107.0
8.2		8.5	
8.3		8.3	
Mean . . . 8.39		Mean . . . . 8.34 Mean . . . 30 observations . . 8.36	

Redetermination of Zero Reading of Instrument

	Volts		
Slit No. 1, 50.0	107.0	Slit No. 2, 52.3	Slit No. 2, 51.3
Slit No. 2, 52.6		52.3	51.3
51.3		52.3	50.3
52.0		51.3	50.5
51.5		52.2	50.3
52.2		53.0	51.6
50.3		52.7	51.1
52.3		51.8	50.3
52.4		51.9	51.2
53.2		52.3	50.0
53.0		51.9	

Mean of 30 observations	51.36
First reading	51.30
Mean	51.33

Calculation of the absorption factors for red light, $0.661\ \mu$.

Let the reading of slit No. 1 be S_1 and slit No. 2 be S_2 when no glass is interposed between lamp and slit No. 2. Also let S_1' and S_2' be the corresponding readings when a glass is inserted; then the absorption factor R is

$$R=\frac{S_1'}{S_2'}\cdot\frac{S_2}{S_1}=\frac{S_1'}{S_1}\cdot\frac{S_2}{S_2'}$$

Glass No. 1 . . . . $R_1=\frac{100}{50}\cdot\frac{51.3}{8.36}=12.3$

Glass No. 2 . . . . $R_2=11.8$

Glass No. 3 . . . . $R_3=11.9$

whence

$$R=R_1R_2R_3=1725\ \text{(red)}\ .$$

The absorption factors for the green and blue glasses, obtained in the same manner, are 340 and 656 respectively.

The lamps used for estimating the sun's temperature were calibrated in a small telescope of 2.68 cm aperture. The distance from the filament to the aperture was 59.8 cm. In the observatory telescope the distance from filament to aperture was 157.5 cm and the aperture was 1.49 cm in diameter.

The ratio of the two solid angles gives the reduction factor for the telescope. We therefore obtain:

$$R'=\frac{\pi}{4}\cdot\frac{(2.68)^2}{(59.8)^2}\div\frac{\pi}{4}\cdot\frac{(1.49)^2}{(157.5)^2}=22.3\ .$$

The resultant reduction factors for the three colors then become:

$$\text{For } 0.661\ \mu \quad R=22.3\times1725=38{,}500 \tag{1}$$

$$0.537\ \mu \quad R=22.3\times\ 340=7580 \tag{2}$$

$$0.446\ \mu \quad R=22.3\times\ 656=14{,}610 \tag{3}$$

3. *Temperature measurement of the sun's disk.*—The distance across the sun's disk was measured by means of a micrometer scale in the finder of the telescope, but the number corresponding to the center of the disk would of course change if the lamp were raised or lowered. For the observations carried out for the red and green light 54 corresponded to the center of the disk and 69 to the extreme edge or limb.

The radius of the disk is thus equal to 15 divisions on the scale of the finder. In the following tables the readings of the ammeter are given for various distances from the center of the disk. When the filament was adjusted to the desired point on the disk the current through the filament was varied continuously until the tip had the same intrinsic intensity as the region surrounding it or until it disappeared against the disk.

The following (Table V) is a sample of the data obtained for the variation of the temperature with distance from the edge to the center of the disk.

TABLE V

AMMETER READINGS FOR GREEN LIGHT

$\lambda = 0.537\,\mu$

69	68	67	64
80.0	84.0	86.0	91.0
82.0	84.5	87.5	91.5
79.5	83.5	87.0	89.5
81.0	83.5	86.5	89.0
81.0	83.5	88.5	91.0
81.0	83.0	87.5	91.5
81.0	85.0	87.5	89.5
82.0	82.5	86.5	90.5
81.5	82.5	87.5	91.5
81.0	82.5	85.5	90.0
Mean....81.1	Mean.....83.5	Mean.....87.0	Mean.....91.0

60	54	69	69	69
92.0	93.5	81.5	80.0	80.0
93.0	92.5	82.0	80.5	80.0
91.5	93.5	81.5	81.5	81.5
91.5	93.0	82.0	80.5	80.0
93.0	91.0	81.5	80.0	80.5
93.0	92.0	81.5	80.0	82.0
92.5	92.0	81.0	82.0	80.0
91.5	92.0	82.0	80.0	80.5
92.5	91.5	81.5	81.0	81.0
91.0	91.5	81.5	81.5	80.0
Mean.92.2	Mean.92.3	Mean of 30 observations.............81.0		

In this case the observation on the edge, i.e., 69, was repeated and it is seen that the agreement is better than might be expected considering the uncertainties of such measurements.

Instead of reducing these readings to temperatures of the sun's disk, a curve was plotted for each color, co-ordinating ammeter readings and distances from center of disk. For any particular distance, the corresponding ammeter reading may be obtained directly from the curve. This gives a better average of all the values taken across the disk.

4. *Reduction of an observation.*—Since it is somewhat easier to use Wien's formula for the reduction of these temperatures, that formula will be used for all the calculations. A temperature estimation will also be made by means of Planck's formula to show the difference in the two results.

We shall consider in detail only the data obtained for red light, as the same method applies equally well to green and blue.

Let T' equal the black-body temperature of the sun's disk, E' the intensity of radiation incident upon the objective of the telescope. Also let T be the apparent temperature of the sun's image and E the intensity of the energy transmitted by the absorbing media of the telescope.

Wien's formula may now be written for the two cases as follows:

$$\log E' = k_1 - k_2 \frac{1}{T'} \tag{4}$$

and

$$\log E = k_1 - k_2 \frac{1}{T} . \tag{5}$$

Subtracting (5) from (4) we obtain:

$$\log \frac{E'}{E} = k_2 \left(\frac{1}{T} - \frac{1}{T'} \right) .$$

But

$$\frac{E'}{E} = R$$

where R is the reduction factor of the telescope.

Whence

$$\frac{1}{T} - \frac{1}{T'} = \frac{\log R}{k_2} = k = \frac{\log R\lambda}{14{,}500 \times 0.4343} .$$

From (1):

$$R = 38{,}500$$

and

$$\lambda = 0.661 .$$

Therefore

$$k=\frac{\log 38{,}500\times 0.661}{14{,}500\times 0.4343}=0.000482$$

whence

$$\frac{1}{T'}=\frac{1}{T}-0.000482\,. \tag{6}$$

Now consider curve 1, Fig. 4, for scale division 54, i.e., the center of the sun's image; the ammeter reading is 67.2, and this corresponds to a temperature of 1317° C. or 1590° A. If we substitute this value for T in equation (6) we obtain for T' the

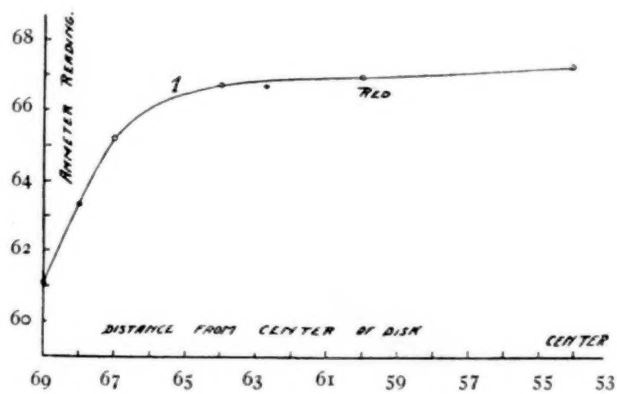

FIG. 4

temperature of the sun's disk, a value of 6803° A. In this manner data were obtained for curves 2, 3, and 4, Fig. 5.

As we move from the center of the disk toward the limb, the temperature falls off more rapidly for the shorter wave-lengths, but near the limb it falls off less rapidly.

There has always been considerable discussion as to the best value of the constant C_2. The value used by Lummer, Pringsheim, Paschen, and Wanner is 14,500. Our own Bureau of Standards[1] also accepts the same value but the value determined by

[1] *Bulletin Bureau of Standards*, **3**, No. 2.

Holborn and Valentiner is much lower 14,200.[1] Again, Nernst and Wartenberg use the value 14,600.[2] To show the effect of the

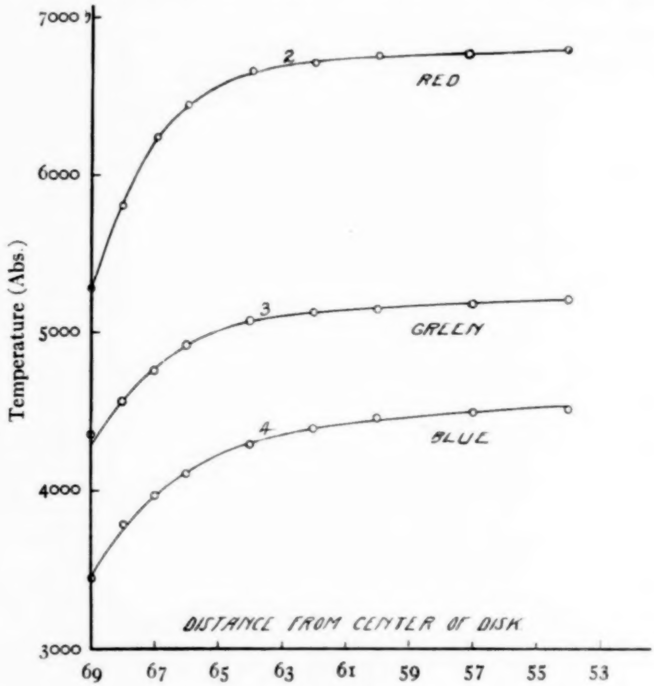

FIG. 5

variation of this constant on the value of the temperature of the sun, the following values were calculated for red and blue light:

TABLE VI

C_2	Red 0.66 μ T'	Blue 0.446 μ T'
14,000	6135	4310
14,100	6211	4348
14,200	6329	4386
14,300	6494	4425
14,400	6667	4464
14,500	6803	4505
14,600	6944	4545
14,700	7143	4587
14,800	7299	4630

[1] *Ann. der Physik*, **22**, 1, 1907.

[2] *Verh. der deutschen phys. Ges.*, **8**, 48, 1906.

We shall now determine the value of the temperature for red light $\lambda = 0.661\ \mu$ by means of Planck's formula in order to see what error results by using Wien's formula.

Using the same notation, we may write Planck's formula for the two temperatures as follows:

$$E' = C_1 \lambda^{-5} \frac{1}{\left(e^{\frac{c_2}{\lambda T'}} - 1\right)} \tag{7}$$

and

$$E = C_1 \lambda^{-5} \frac{1}{\left(e^{\frac{c_2}{\lambda T}} - 1\right)}\,. \tag{8}$$

Dividing (8) by (7) we obtain:

$$\frac{E'}{E} = \frac{\left(e^{\frac{c_2}{\lambda T}} - 1\right)}{\left(e^{\frac{c_2}{\lambda T'}} - 1\right)} = R\,. \tag{9}$$

Writing (9) in the form:

$$\left(e^{\frac{c_2}{\lambda T}} - 1\right) \frac{1}{R} + 1 = e^{\frac{c_2}{\lambda T'}}$$

we finally obtain:

$$T' = \frac{\frac{c_2}{\lambda}}{\log \left[\frac{1}{R} \left(e^{\frac{c_2}{\lambda T}} - 1\right) + 1 \right]}\,. \tag{10}$$

Now let

$$\frac{C_2}{\lambda} \log\ e = k_2$$

whence

$$T' = \frac{k_2}{\log \left[\frac{1}{R} \left(e^{\frac{k_2}{\lambda T}} - 1\right) + 1 \right]}\,. \tag{11}$$

To evaluate $e^{\frac{k_2}{\lambda T}}$ let

$$\frac{k_2}{\lambda T} = \log x$$

whence

$$\log x = \frac{14{,}500 \times 0.4343}{0.661 \times 1590} = 5.99$$

and

$$x = 977{,}000\,.$$

Since

$$R=38{,}500$$

$$\log\left[\frac{977{,}000}{38{,}500}+1\right]=\log\,(25.3+1)=1.420.$$

The constant k_2 is 9520 and the real temperature now becomes:

$$T'=\frac{9520}{1.420}=6700^\circ\text{ A}.$$

If we neglect the 1 in the expression $\log(25.3+1)$, then the expression reduces to Wien's formula in which case

$$\log 25.3=1.403$$

whence

$$T=\frac{9520}{1.403}=6800^\circ\text{ A}.$$

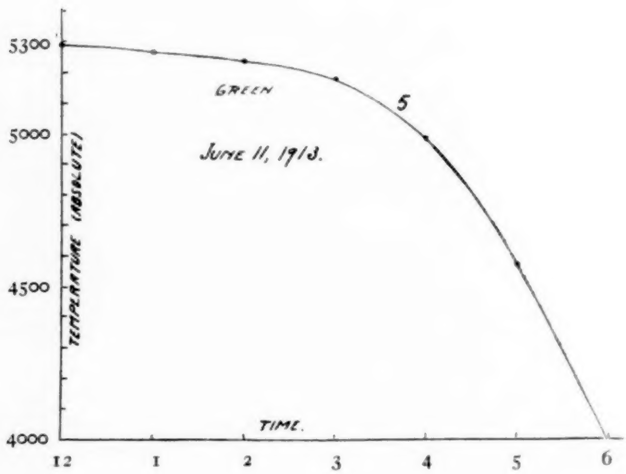

FIG. 6

The variation of the apparent temperature with the sun's zenith distance was determined for green light and the results are shown in curve 5, Fig. 6.

5. *Correction for atmospheric absorption.*—The ratio of the intensity of the sun, at the boundary of the atmosphere and at the surface of the earth, is given by the well known formula:

$$\frac{E_s}{E_e}=\frac{1}{A^{\sec Z}} \tag{12}$$

where

E_s = intensity of sun at the boundary of the atmosphere

E_e = intensity of sun at the telescope

A = the transmission coefficient

Z = the zenith distance

If the intensity is known for two different hours, say 12:00 and 4:00 o'clock, then A may be determined from the relation:

$$\log A=\frac{\log \frac{E_{12}}{E_4}}{\sec Z_{12}-\sec Z_4}. \tag{13}$$

By means of curve 5, the apparent temperature of the sun for these two hours may be determined and by means of Wien's formula the ratio of the corresponding intensities may be determined. Writing Wien's formula for the two cases we obtain:

$$\log E_{12}=k_1-k_2\frac{1}{T_{12}}$$

and

$$\log E_4=k_1-k_2\frac{1}{T_4}$$

whence

$$\log \frac{E_{12}}{E_4}=k_2\left(\frac{1}{T_4}-\frac{1}{T_{12}}\right). \tag{14}$$

By means of (14), using the data obtained from curve 5, we obtain:

$$\log \frac{E_{12}}{E_4}=0.14$$

and by means of tables

$$Z_{12}=17.1 \text{ and } Z_4=54.3$$

whence

$$\log A = \frac{0.14}{\sec 17.1 - \sec 54.3} = 0.803 - 1$$

and

$$A = 0.63 .$$

The mean of 5 values of A determined for 2:00, 3:00, 4:00, 5:00, and 6:00 o'clock was found to be 0.64.

Using the values of the transmission coefficients obtained for Washington and Mount Wilson[1] the following values were found by interpolation for red and blue light:

For red light, 0.661 μ, $A = 0.74$, and for blue light, 0.446 μ, $A = 0.50$.

The absorption factor already determined for the telescope will now be determined for red light, 0.661 μ.

Equation (12) may be written in the form

$$\log \frac{E_s}{E_e} = -\sec Z \log A .$$

The time of observation of the temperature for red light was 2:00 P.M., September 1, 1912, and the zenith distance for this hour is 42°.7. We therefore obtain:

$$\log \frac{E_s}{E_e} = -\sec 42.7 \times \log 0.74 = 0.178$$

and

$$\frac{E_s}{E_e} = 1.505 .$$

The reduction factor for red light, 0.661 μ, corrected for atmospheric absorption now becomes:

$$R' = 38{,}500 \times 1.505 = 58{,}000$$

and the new constant k' is

$$k' = \frac{\log 58{,}000 \times 0.661}{14{,}500 \times 0.4343} = 0.000498 .$$

[1] Abbot, *The Sun*, p. 242.

Equation (6) therefore becomes:

$$\frac{1}{T'}=\frac{1}{T}-0.000498$$

and the corrected temperature for the center of the disk is 7580° A. In a similar manner the reduction factors for the other colors were found to be 15,300 for green and 37,000 for blue light. The data for green light were determined on September 22, 1912, at 2:00 P.M., and that for blue light on April 15, 1913, at 2:00 P.M.

The sun's temperature for the three colors is therefore as follows:

For red light, 0.661 μ: 7580° A.
For green light, 0.537 μ: 5990° A.
For blue light, 0.446 μ: 5230° A.

Without the absorption of the light through the atmosphere of the earth we find for the three colors the following values:

For red light 6803° A.
For green light 5208° A.
For blue light 4505° A.

There is possibly a small error in the determination of the transmission coefficients of the atmosphere, due to the fact that these coefficients were not determined at the same time and therefore possibly not under exactly the same conditions of the atmosphere as those under which the radiation of the sun was measured.

The last two series of values indicate clearly that the sun is not a black body, because for a black body we should find the same temperature for each wave-length. This fact is also demonstrated directly by the actual distribution of the energy of radiation through the spectrum and through the results of the three general methods which may be used for the determination of the sun's temperature based upon the following laws:

Stefan-Boltzman Radiation Law:

$$E=76.8\times10^{-12}T^4,$$

Wien's Displacement Law:

$$\lambda_m T=2930.$$

Planck's Distribution Law:

$$E=e_1\lambda^{-5}\frac{1}{\left(e^{\frac{c_2}{\lambda T}}-1\right)}\,.$$

The first method gives a temperature of 5830° A. if we use Abbot's value of the solar constant 1.922. In the second method, if we take the wave-length of the maximum energy as 0.470 μ[1] we get a temperature of 6230° A. The third method, as has just been shown by the present investigation, gives a temperature of 7580° A. All the determinations of the temperature of the sun by means of radiation give therefore only approximative results, and the deviations of the temperatures for different colors and for different methods indicate how far the sun's radiation differs from that of a black body. Another example taken from laboratory practice may illustrate the difference between the thermodynamic temperature of a radiating body and the temperature obtained by radiation methods. If we attempt to measure the temperature of a piece of iron at about 1600° A., by means of the total radiation emitted we obtain a temperature which is about 400° lower than the true temperature, but if we utilize the radiation corresponding to a single wave-length, say 0.6 μ, we obtain a temperature which is about 150° lower.

SUMMARY

1. The temperature of the sun has been measured by a new method based on Planck's and Wien's laws of radiation for three different wave-lengths.

2. The variation of the radiation of the sun from the center to the limb has been measured for three different colors.

3. The absorption of green light in the atmosphere of the earth has been measured.

In conclusion I wish to thank Professor A. P. Carman and Professor J. Kunz for their many helpful suggestions during the investigation of the above problem.

Laboratory of Physics
University of Illinois
October 1913

[1] Abbot, *The Sun*, p. 69.

ON THE THEORETICAL PHOTOMETRY OF DIFFUSE REFLECTION

By L. GRABOWSKI

By diffuse reflection is understood the property of a body whereby in contrast to bodies with a polished surface it acts under the influence of radiation as if each element of its surface would send out light to *all* directions of external space; in so doing the intensity J of this apparent luminosity toward the different directions of external space (different directions of emanation) follows a law of the form

$$J=\delta \cdot F(i, \epsilon, \eta) . \tag{1}$$

Here δ signifies the intensity of the incident luminous radiation at the point of the surface under consideration (spatial density of the luminous energy), and i denotes the angle of incidence of these rays. ϵ denotes the angle of emanation, and η the azimuth of this direction reckoned in the tangential plane from the plane of incidence (azimuth of emanation). F is a function peculiar to the body.

In what follows we shall denote this phenomenon, which has been hitherto called diffuse scattering or diffuse reflection, as briefly "difflection." *Per contra*, we shall use the simple term "reflection" to denote the phenomenon ordinarily called regular reflection. This is characterized by the fact that J differs from zero only for a single direction of emanation, namely, for $\eta=180°$ and $\epsilon=i$. This value is $J=\delta \cdot f(i)$, where f is a function peculiar to the body. According to Fresnel's law of reflection (the correctness of which we shall not presuppose in what follows) the function f has, as is well known, for all polished bodies the following form:

$$f(i)=\frac{1}{2}\left[\frac{\tan^2(i-r)}{\tan^2(i+r)}+\frac{\sin^2(i-r)}{\sin^2(i+r)}\right], \quad \sin r=m \sin i$$

where m is a constant of the body (the reciprocal of the index of refraction).

The existing theories of "difflection" concern themselves almost exclusively with the case where the function of "difflection" F in

equation (1) does not contain the azimuth of emanation η, so that it is reduced to $F(i, \epsilon)$. The apparent emission of light from an element of surface of such a body is therefore equally strong for all directions of emanation which form a circular cone about the normal. We shall say in this case that the difflection is circular. Several different expressions have been proposed for the form of the function F, of which the most important for theoretical investigations are the law of Lambert and that of Lommel-Seeliger. The first of these constitutes merely an assumption not thoroughly founded, the latter is analytically derived from certain plausible conceptions of the cause of the phenomenon of difflection. The first contains one undetermined constant, the second two. Experimental investigations on different difflecting substances have, however, shown that in the first place many of them do not "difflect" in a circular manner at all, and, second, that even in circular difflection neither the Lambert, nor the Lommel-Seeliger, nor any other law has unlimited validity, but rather that different substances follow different laws of difflection.

Bouguer formed a conception of the physical cause of difflection which unquestionably must appear at the first glance as a thoroughly plausible explanation of the phenomenon. He assumes that each element of surface consists of countless infinitely small mirrors which are pointed toward all possible directions. H. von Seeliger has more recently tested Bouguer's hypothesis analytically and established the fact that it is impossible to determine the frequency function of the orientations of the mirrors, and the law of reflection f which holds for the mirrors so that there shall result a difflection according to Lambert's or according to the Lommel-Seeliger law.

It will be proved in what follows that the phenomenon of circular difflection cannot be explained by the Bouguer hypothesis, *whatever may be the special law of difflection of the given body* (excluding the law $F(i, \epsilon) = \text{constant}$).

Imagine around the position (P) of an element of surface df a sphere constructed with any selected large radius and mark on this spherical surface by the points N, S, O (Fig. 1, view from above), the directions: of the external normal of the element, the direction

meeting the incident rays, and the direction toward the point of observation. If *PM* is the direction which falls just in the middle between the two last directions, then it is clear that of all the countless little mirrors of which *df* is constituted, only those can send light to the point of observation whose normals have just the direction *PM*. If we designate the number of little mirrors of this orientation

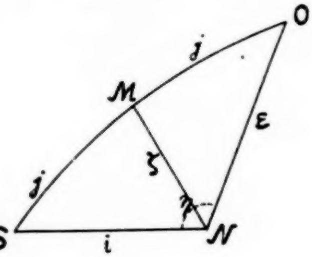

Fig. 1

contained in the element *df* by *ndf*, then we shall have to assume that *n* is a function (unknown) of the zenith distance ζ of the direction *PM*, but is independent of the azimuth of this direction. If the average area of a mirror is σ, then we may place for the total surface $ndf \cdot \sigma$ of mirrors oriented toward *M:*

$$\phi(\zeta) \cdot df .$$

ζ may be expressed according to formulae of spherical trigonometry by the angle of incidence i of the rays with the normal of *df*, the angle of emergence ϵ, and the azimuth of emergence η. We have:

$$\cos 2j = \cos i \cos \epsilon + \sin i \sin \epsilon \cos \eta \tag{2}$$

and (from the formulae for cos i and cos ϵ from the two triangles):

$$\cos \zeta = \frac{\cos i + \cos \epsilon}{2 \cos j} = \frac{\cos i + \cos \epsilon}{\sqrt{2}\sqrt{1 + \cos i \cos \epsilon + \sin i \sin \epsilon \cos \eta}} . \tag{3}$$

The angle of incidence of the rays in respect to the normal to the effective mirrors is j. If we designate the function of reflection by the symbol f, and if we assume Fresnel's law, we should put

$$f(j) \equiv \tfrac{1}{2} \left\{ \frac{\tan^2 [j - \text{arc sin } (m \sin j)]}{\tan^2 [j + \text{arc sin } (m \sin j)]} + \frac{\sin^2 [j - \text{arc sin } (m \sin j)]}{\sin^2 [j + \text{arc sin } (m \sin j)]} \right\}$$

then the intensity of the apparent luminosity of the element in the direction *PO* will be given by the expression

$$J_{\epsilon,\eta}=\delta\cdot\phi(\zeta)\cdot f(j)\,, \tag{4}$$

where ϕ is an unknown function, while ζ and j are expressed by equations (3) and (2) in terms of i, ϵ, η.[1]

If the phenomenon of pure circular diffiection should arise, expression (4) would remain unchanged, if O goes in a circle about N. Hence the following relation must exist between the functions ϕ and f,

$$\phi(\zeta)=\frac{\phi(0)}{f(0)}f(\zeta)\,,$$

which is found by comparison of the two values of J [$\delta\cdot\phi(0)\cdot f(i)$ and $\delta\cdot\phi(i)\cdot f(0)$] for $\eta=180°$, $\epsilon=i$, and for $\eta=0$, and $\epsilon=i$. Expression (4) therefore is transformed into:

$$J_{\epsilon,\eta}=\frac{\phi(0)}{f(0)}\delta\cdot f(\zeta)\cdot f(j)\,. \tag{5}$$

The necessary and sufficient condition for pure circular diffiection in respect to the reflection function reads, *the reflection function f must be so constituted that the expression*

$$\frac{\phi(0)}{f(0)}\cdot f(\zeta)\cdot f(j) \tag{6}$$

is a function of ϵ *alone for every given* $i<90°$, *but is independent of* η (ζ and j being expressed in terms of i, ϵ, η, by means of equations (3) and (2)). (In this theorem the case of $i=90°$ is excluded because the relation $(\phi)0\cdot f(i)=\phi(i)\cdot f(0)$, which was used in the derivation of this theorem, resulted from the consideration that for a point of observation symmetrically opposed to S with respect to N, only those mirrors are effective whose normals are directed toward N; but this is not correct in the case $i=90°$, for then all those mirrors send light to the point of observation, the normals of which are directed to points of the vertical circle perpendicular to NS.)

In our further discussion we shall limit ourselves, as is sufficient for our negative proof, to the consideration of the case that the

[1] The formulae developed to this point have already been given by von Seeliger. Their derivation is reproduced here for the convenience of the reader.

point O goes around N on the circle ***passing through*** S, whence $\epsilon = i$. ζ and j will then on account of their dependence on the azimuth of emanation vary as indicated by the following equation in which s denotes $|\sin i|$, and instead of the azimuth of emanation the new variable has been introduced, $\alpha = \left|\sin \frac{\eta}{2}\right|$:

$$\left.\begin{aligned} \sin j &= s\alpha\,, \\ \sin \zeta &= \frac{s\left|\sqrt{1-\alpha^2}\right|}{\left|\sqrt{1-s^2\cdot\alpha^2}\right|}\,. \end{aligned}\right\} \quad (7)$$

In order that the intensity of illumination shall be the same in all directions which have the same inclination to the normal of the element as the direction of the incident ray, the function f must have the property that, when (7) is introduced in the expression (6), α is canceled; hence $f(\zeta)f(j)$ must then become a function of s alone. This function, however, as the consideration of the special case $\alpha = 1$ (O symmetrical to S) teaches, is nothing other than $f(0)f(i)$. Since we are concerned in every reflection function $f(i)$ only with values of the argument between 0 and $\frac{\pi}{2}$, we may regard each given reflection function as also a function of $\sin i$, by setting $f(i) \equiv U(\sin i)$. Our condition therefore reads that the reflection function must have the property

$$U\left(\frac{s\left|\sqrt{1-\alpha^2}\right|}{\left|\sqrt{1-s^2\cdot\alpha^2}\right|}\right)\cdot U(s\alpha) = U(0)\cdot U(s)\,, \tag{8}$$

and this identity must be fulfilled for all values of $s(0 \leqq s < 1)$ and for all values of $\alpha(0 \leqq \alpha \leqq 1)$.

We shall now seek for the general solution of this functional equation. If we differentiate (8) partially, first with respect to s and then with respect to α, and if we subtract the second equation after multiplication by $\frac{\alpha}{s}$ from the first equation, we obtain:

$$U'(q_{s,\alpha})\left[\frac{\partial q_{s,\alpha}}{\partial s} - \frac{\alpha}{s}\,\frac{\partial q_{s,\alpha}}{\partial \alpha}\right]\cdot U(s\alpha) = U(0)\cdot U'(s)$$

in which we have temporarily abbreviated by placing

$$\frac{s\left|\sqrt{1-\alpha^2}\right|}{\left|\sqrt{1-s^2\alpha^2}\right|} = q_{s,\alpha}\,.$$

If we introduce here for $U(s\alpha)$ its value from (8), and carry out the differentiation indicated in the square brackets, we get:

$$\frac{U'(q_{s,\alpha})}{U(q_{s,\alpha})} \cdot \frac{1}{\sqrt{(1-s^2\alpha^2)(1-\alpha^2)}} = \frac{U'(s)}{U(s)}.$$

If we now introduce a new functional symbol Ω defined by

$$x\frac{d \log U(x)}{dx} \equiv \Omega(x)$$

we shall finally obtain:

$$\Omega\left(\frac{s\sqrt{1-\alpha^2}}{\sqrt{1-s^2\alpha^2}}\right) = (1-\alpha^2) \cdot \Omega(s) \tag{9}$$

as the condition which the sought-for function Ω shall satisfy for all values of $s(0 \leqq s < 1)$ and all values of $\alpha(0 \leqq \alpha \leqq 1)$.

This equation has in fact a solution for the function Ω.[1] In order to find this function we differentiate (9) logarithmically, once partially with respect to s, the next time partially with respect to α. Thus we find, if we temporarily set $\log \Omega(x) \equiv W(x)$, that the function W' must satisfy *both* conditions (wherein $q_{s,\alpha}$ is the same abbreviation as before): first,

$$q_{s,\alpha} W'(q_{s,\alpha}) = sW'(s) \cdot (1-s^2\alpha^2);$$

and on the other hand

$$W'(q_{s,\alpha}) = \frac{2}{q_{s,\alpha}(1-q^2_{s,\alpha})};$$

or, in general,

$$W'(x) = \frac{2}{x(1-x^2)},$$

where x may represent any value whatever between 0 and 1 $(0 \leqq x < 1)$. From the last equation it follows that $W(x)$ must equal

$$\log C\left(\frac{x^2}{1-x^2}\right)$$

[1] It may be incidentally remarked that a functional equation of this sort does not necessarily have a solution, but on the contrary it will generally not have one. Thus equation (9) would be impossible if in the right-hand member under the symbol Ω instead of s, for instance s^2, or some other definite function of s (multiples of s excepted), should occur.

(C being an undetermined constant); whereby the first condition is fulfilled. The solution of equation (9) is therefore:

$$\Omega(x)=\frac{Cx^2}{1-x^2}\,.$$

Now since

$$\log U(x)\equiv\int\frac{\Omega(x)}{x}dx\,,$$

we finally obtain, as the general solution of the functional equation (8)

$$U(x)=\frac{A}{(1-x^2)^n}\,,$$

where A, n are two undetermined constants.[1]

To return now to the reflection function f, we must place $x=\sin i$, and we obtain as the law of reflection

$$f(i)=A\ \sec^{2n}i=f(0)\cdot\sec^{2n}i\,. \tag{10}$$

This law of reflection not only disagrees with that of Fresnel, but it is a priori impossible, at least if we disregard the special case $n=0$, or $f(i)=$constant; and if we further demand that the reflection function shall be an increasing one. This last condition is justified by the consideration that the reflection function must take the

[1] The solution of the functional equation (8) can be accomplished in a somewhat shorter way, as Professor von Seeliger kindly indicated to me in a letter after reading this paper. Putting in general $U(x)=F(1-x^2)$, and $1-s^2=u$, $1-s^2\alpha^2=v$, (8) becomes

$$F\left(\frac{u}{v}\right)=\frac{F(u)}{F(v)}\cdot F(1)\,. \tag{a}$$

By logarithmic differentiation of this equation with respect to u and the application of the equation thus obtained to the special case of $\alpha=1$, that is, $u=v$, it is easily found that

$$u\frac{F'(u)}{F(u)}=\frac{F'(1)}{F(1)}\;;$$

therefore if we place

$$\frac{F'(1)}{F(1)}=-n,$$

it follows that

$$F(u)=A\cdot u^{-n}\;;$$

A and n are arbitrary, as will be seen by introducing (b) in (a). The expression for function U found in the text follows from (b) immediately.

value 1 for $i=\frac{\pi}{2}$, while, on the other hand, it may not be larger than 1 for any value of i. If the function (10) shall be an increasing one, then the constant n must evidently be positive. But then with increasing i, the right-hand member increases *without limit* and must therefore with a definite value of i become greater than 1. A mirror would, therefore, with sufficiently oblique incidence reflect more light than it received; and the intensity could be indefinitely increased by inclining the mirror, an obviously impossible conception.

But if we regard as valid the law of reflection $f(i)=$ constant, then equation (4) does show that in this case circular difflection can occur; and as may readily be seen by applying (4) with $f(j)=$ constant to the motion of a point O about any circle around N, it will occur always and only if the $\phi(\zeta)=$ constant, that is, if an equal number of normals to the mirror point to each external direction of space. Equation (5) further shows that the circular difflection arising in this case is equal in all directions, and, moreover, that it is independent of the angle of incidence of the radiation illuminating the element with the normal to the element.

We may therefore say in summarizing: *No given law of circular difflection can be explained on the hypothesis of countless small mirrors*, with the exception of the law of "difflection" $J=\delta\cdot$ constant. This special case requires, however, for such an explanation, the assumption that the reflection at a mirror is independent of the angle of incidence with the normal to the mirror, and, further, that the surface of the illuminated element has such a structure that an equal number of normals to the mirror point to every direction of external space.

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PHOTOGRAPHIC PHOTOMETRY WITH THE 60-INCH REFLECTOR OF THE MOUNT WILSON SOLAR OBSERVATORY[1]

By FREDERICK H. SEARES

I. INTRODUCTION

The following is an account of the methods used for investigations in photographic photometry with the 60-inch reflector of the Mount Wilson Solar Observatory. It includes a statement of the underlying principles and the processes used to test their reliability, a description of the procedure for the measurement and reduction of the photographs, and a general indication of the precision of the results.

The size of the instrument fixes its field of greatest usefulness among the fainter stars; the large ratio of aperture to focal length (1 to 5) and the consequently limited field restrict its application to isolated regions or to relatively small areas of the sky. It is possible to secure photometric results in such abundance as to be of service for statistical investigations, but anything approaching a photometric survey would be inadvisable.

One of the most important applications of the instrument, and that with which this paper is principally concerned, is the determination of faint standards of magnitude. This imposes the condition that the methods employed shall be such as to provide reliable determinations of the scale. For the faint stars modification is required according as the exposures are moderate or long, and the desirability of comparing the scale for the fainter objects with that of the bright stars leads to further adaptation. We have thus to deal with three classes of objects: bright stars, whose lower limit is at the 10th magnitude; intermediate stars, including those between magnitudes 10 and 18; and faint stars, from the 18th magnitude to the faintest that may be registered with long exposure.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 80.

All the methods involve successive exposures upon the same plate. For the bright and intermediate stars these are of the same duration, one being made with the full aperture, the other with the intensity reduced by a known amount. The comparison of the images for the full and the reduced intensities leads, in the case of the intermediate stars, to the relation connecting size of image and magnitude which fixes the scale. For the bright stars it gives at once an extension of a scale assumed to have been previously established for the intermediate group. For the third class—the faint stars—the method involves again an extension of a known scale; but in this case the results are based on an application of the law of photographic action.

2. THE METHODS

a) Intermediate stars.—These are observable with what may be regarded as a normal arrangement of the program. Neither the intensity reduction nor the exposure required is extreme. Errors resulting from the use of large reduction constants are thus avoided, and, at the same time, the maximum exposure, which may be set at 30 or 40 minutes, is short enough to insure reasonable freedom from atmospheric disturbances. As an extension to either the brighter or the fainter objects presupposes a knowledge of the scale for the intermediate stars, the methods applicable to this class are considered first.

If I and I' represent the maximum and the reduced intensities, the reduction in magnitudes is

$$\Delta m = 2.5 \log \frac{I}{I'} . \qquad (1)$$

Let $i_1, i_2, i_3 \dots$ denote the full aperture images arranged in order of decreasing size; and, similarly, let $i'_1, i'_2, i'_3 \dots$ be the corresponding reduced-intensity images. The differences

$$i_1 - i'_1, \quad i_2 - i'_2, \quad \dots \; i_n - i'_n, \quad \dots \; i_r - i'_r, \quad \dots$$

expressed in some convenient unit, correspond to the magnitude interval Δm. We now make the important assumption that S_n, a star Δm magnitudes fainter than S_1, produces during the maximum-intensity exposure the same photographic effect as S_1

during the reduced-intensity exposure. By this assumption, $i_n = i'_1$. Similarly, if the star S_r be Δm magnitudes fainter than S_n, we have $i_r = i'_n$. In practice, the matter is reversed. If we observe the equalities of images indicated, we conclude that S_1 and S_n, S_n and S_r differ by Δm magnitudes. Let the brightness of S_1 be M. The magnitudes of S_n and S_r are therefore $M+\Delta m$ and $M+2\Delta m$, respectively, and we have established a scale for the stars in question. The scale thus derived is absolute, for the relation between star-intensity and magnitude is independent of assumed magnitudes. Usually the equality of images indicated is not exact; but the inclusion of this circumstance and the extension of the scale to other stars afford no difficulty. Both are accomplished by an interpolation process described in a later section.

To reduce the results to the International System, they must be referred to the standard zero point. This is defined by the Harvard magnitudes of stars of spectrum A0 whose brightness falls within 5.5 and 6.5 of the Harvard scale. The reference involves, in some form, a comparison of the calculated magnitudes with others already based on the International System. This may be variously accomplished, but the principles involved are so simple that they require no description.

We now consider the fundamental assumption: if the primary image i_n of a faint star equals the secondary image i'_1 of a brighter star, the two differ by Δm magnitudes, Δm being defined by equation (1), in which I and I' are the intensities active during the primary and secondary exposures, respectively.

As far as photographic phenomena are concerned, there appears no reason for doubt. The assumption is that equal photographic effects produced during equal exposure times must be caused by equal light-intensities. Disregarding local variations in sensitiveness, it is difficult to find an objection, although a question is perhaps raised by the phenomenon described below. The fact that the exposure times are equal avoids difficulties which would otherwise enter.

There are, however, at least two circumstances which may invalidate in some degree the fundamental assumption. One arises from the fact that most methods of reducing the intensity

involve a change in the aperture of the instrument; the other has its origin in atmospheric disturbances.

Any change in the aperture entails a corresponding modification in the diffraction pattern of the optical image. If the aperture be reduced by one-half, the diffraction disk and its surrounding rings will be doubled in diameter; and it is not certain that equal quantities of light distributed over unequal diffraction images produce the same photographic effect. Nor, conversely, is it possible to infer from the equality of the images i_1' and i_n that they have been produced by equal quantities of light. The question is one requiring special investigation before any method which involves a change in the diffraction pattern can be accepted as unobjectionable. It is discussed in the section on the diffraction effect.

The relation of atmospheric conditions to the fundamental assumption is obvious. Changes in transparency during either or both of the exposures modify the relative size of primary and secondary images. A similar disturbance may enter through irregularities in seeing. If these be greater during one exposure than the other, systematic differences between the images produced by equal quantities of light will occur. Both factors vitiate the determination of the scale by an amount which may be large or small according to circumstances. The difficulties are inherent in any method requiring successive exposures, but with a powerful instrument they are minimized by the relatively short exposures. With the 60-inch reflector, 10 or 15 minutes reaches the 17th photographic magnitude, and the trouble is not serious. Moreover, a repetition of the exposures in the reverse order favors the elimination of minor disturbances, such as those arising from changing zenith distance, and affords a test which reveals the presence of irregularities that are excessive.

Aside from the diffraction effect and the influence of atmospheric conditions, there is a third source of error which may affect results derived from successive exposures. It relates to a phenomenon noted by Pickering.[1] When several exposures are impressed

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 64, p. 7; *Astrophysical Journal*, **36**, 374, 1912.

on the same plate, the first and last being short and equal, the images of the first are often systematically brighter than those of the last, the differences amounting on the average to a quarter of a magnitude. The cause is obscure, though apparently photographic in origin. Although measures at the Harvard Observatory revealed the effect on some of the earlier Mount Wilson photographs, it does not appear to be appreciable upon others subsequently obtained.[1] Here again, a symmetrical arrangement of the exposures presumably would eliminate the greater part of the error, and, in any event, would serve as a valuable control.

Scale errors arising from the diffraction effect are constant for any given method of reducing the intensity. For wire gauze screens they are zero, since the central diffraction disk is the same with as without the screen. For the diaphragms, they are negligible, as will appear later. Errors caused by atmospheric disturbances, changing zenith distance, and the photographic phenomenon just described are systematic for a plate. The use of short exposures, symmetrically arranged, reduces them to a minimum. As some duplication of plates is necessary, they tend, in the final result, to become accidental.

b) ***Faint stars.***—Atmospheric disturbances set a limit to the exposure times that may be used when successive exposures are employed. The limit depends upon the character of the night. Fifteen or twenty minutes may be used upon any night suitable for photometric work. Little difficulty has been experienced in prolonging the exposures to forty minutes, and upon good nights they may be extended to an hour. But even the maximum limit excludes the possibility of extending the scale to the faintest stars. To meet this difficulty the method now to be described is available.

A knowledge of the scale for the intermediate stars in the same region is presupposed. To extend this to fainter objects, plates of long exposure, preceded or followed by a single short exposure, both with the full aperture, are used. The method involves a change in the energy acting on the plate by a variation in the exposure instead of the intensity. The change is known, but cannot

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 64, p. 12; *Astrophysical Journal*, **36**, 379, 1912.

easily be utilized on account of the complexity of photographic action with different exposures. The reduction constant, therefore, enters as an unknown whose value is to be determined from the stars of known brightness. But the question arises whether, for these conditions, the constant has a specified value. In order that such may be the case, the magnitude difference in the photographic effects produced by the light energies It and It' must be constant, t and t' being the exposure times.

This implies that

$$\Delta m = f(It') - f(It) \tag{2}$$

is independent of I. Let I' be an intensity such that during the second exposure it produces a photographic effect equal to that produced by I during the first. Then

$$f(It) = f(I't'),$$

and by (2)

$$\Delta m = f(It') - f(I't')$$

whence

$$\Delta m = 2.5 \log \frac{I'}{I} . \tag{3}$$

Hence, the constancy of Δm also implies that the ratio of intensities producing the same effects in the two exposures shall be independent of the intensity.

The condition for constant photographic effect is[1]

$$tI \cdot 10^{\alpha} = \text{const.} \tag{4}$$

in which

$$\alpha = -a \sqrt{\log^2 \frac{I}{I_0} + 1} .$$

From (4)

$$\frac{I'}{I} = \frac{t}{t'} 10^{\alpha - \alpha'} \tag{5}$$

in which α' refers to the second exposure. The form of (5) is such that the ratio I'/I cannot be expressed independently of I. Consequently Δm is not constant, but has the form

$$\Delta m = 2.5 \left(\log \frac{t}{t'} + \alpha - \alpha'\right) .$$

[1] *Vierteljahrsschrift der Astronomischen Gesellschaft*, **48**, 120, 1913.

If, however, I and I' are small as compared with I_0, we may write, taking the negative sign of the radical in order that $\alpha-\alpha'$ may be negative (a is assumed to be positive),

$$\alpha = a \log \frac{I}{I_0}, \quad \alpha' = a \log \frac{I'}{I_0}$$

whence

$$\alpha - \alpha' = a \log \frac{I}{I'} .$$

For this case, the condition for constant photographic effect (4) reduces to

$$tI^{1+a} = tI^q = \text{const.} \tag{6}$$

and

$$\Delta m = \frac{2 \cdot 5}{q} \log \frac{t}{t'} . \tag{7}$$

For small values of I and I', the correction to (7) is approximately

$$+\frac{a}{2q} \frac{\Delta m}{\log \dfrac{I}{I_0} \log \dfrac{I'}{I_0}}$$

and decreases as I and I' approach zero. Δm therefore decreases as fainter and fainter stars are considered and approaches (7) as a limit.

Now let

$$\begin{array}{l} \ldots \; i_0, i_1, i_2 \; \ldots\ldots \; i_n, i_{n+1} \; \ldots \; i_r \\ \ldots \; i'_0, i'_1, i'_2 \; \ldots\ldots \; i'_n \end{array}$$

represent the primary and secondary series of images arranged in the order of decreasing brightness, those with the same subscript being of the same star. The problem is to find the magnitudes for the faint images $i_{n+1} \ldots i_r$. For simplicity, suppose that, when written in the form,

$$\begin{array}{rr|ll} \ldots \; i_0, i_1, i_2 \; \ldots\ldots \; i_{n-1}, & i_n, & i_{n+1} & \ldots\ldots \; i_r \\ \ldots\ldots \; i'_0, & i'_1, & i'_2 & \ldots\ldots \; i'_n \end{array}$$

the images in the same vertical line are of the same size. The magnitude difference of any pair of images thus aligned is, by (3), Δm. The absolute magnitudes of the stars o to n are assumed to be known. Denoting them by m with corresponding subscripts,

i.e. magnitudes on an absolute scale

we obtain from the pairs to the left of the vertical line in above arrangement

$$\ldots\ \Delta m_0 = m_{n-1} - m_0\,, \quad \Delta m_1 = m_n - m_1\,. \tag{8}$$

For the faint stars with which we are concerned, (7) is approximately satisfied; the change in Δm is slow and nearly linear, and it is possible to extrapolate for the intensities $i_{n+1}\ \ldots\ i_r$ with some precision. To find the magnitudes for stars $n+1$ to r, we have only to add the extrapolated values of Δm to the magnitudes of stars 2 to n. The method is more precise than an extrapolation of the relation between scale reading and magnitude for the change in m as s increases is much less regular than the variation of Δm.

Since the error in (7) is proportional to the reduction constant Δm, the scale extension should not be too great. The shorter the interval the greater will be the reliability; but if too greatly restricted, the attainment of the lowest limit will become laborious. Two magnitudes, corresponding to a ratio of about 10 to 1 in the exposure times, is a satisfactory value.

c) *Bright stars.*—It is not often necessary to use a powerful instrument for the determination of the magnitudes of bright stars. But for the connection of intermediate stars with the brighter objects it is desirable that a method for an upward extension of the scale should be available. Modification of that described above is necessary, since the images of bright stars are unmeasurable even with very short exposures. Further difficulty is introduced by the fact that the field of the reflector limits the number of bright objects that may be photographed with a single exposure. The required modifications are described in "The Photographic Magnitude Scale of the North Polar Sequence."[1] The method is most conveniently applied when the scale has already been established for an adjacent group of intermediate stars. An exposure is made upon the bright stars with an intensity reduction sufficient to produce images of an apparent brightness falling within the region of known magnitudes. A second exposure, with the full aperture, is then made upon the intermediate stars, and finally, the first exposure is repeated. The subtraction of the reduction constant

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 70; *Astrophysical Journal*, **38**, 241, 1913.

from the apparent magnitudes of the bright stars, found by comparison with the fainter objects, gives the required result. Although the exposures are short (two minutes) the method is tedious on account of the small field of view; but it has been found useful in establishing the scale over a long interval, and has the advantage of giving results that are homogeneous with the adopted scale for the intermediate stars.

It will be seen that some of the difficulties encountered affect all methods involving successive exposures. Although these can be avoided in part by a proper arrangement of the observations, there remains the objection that such methods are wasteful of time. Were it possible to secure the full and reduced intensity exposures simultaneously, the observing time could be reduced by one-half. Such a possibility is afforded by the method of prismatic companions, proposed and used by Pickering.[1] A trial has not been possible, owing to the lack of a suitable prism, which, with the reflector, would have a minimum diameter of 14 inches. It is desirable, however, that the method be employed, both as a control upon processes now in use and because of the saving in time. Its usefulness for the determination of very faint magnitudes is obvious.

Another method with important advantages, especially in the elimination of atmospheric disturbances, involves the use of a screen covering one-half of the plate.[2] Since there is no known absorbing substance which is neutral in tint, wire gauze a little in front of the plate is employed.[3] The method cannot, however, be used advantageously with the reflector. With a grating covering one-half the plate, and at such a distance as to separate the central image from the spectra, the penumbral region is so large as to include a considerable fraction of the area of good definition.

3. MEANS USED FOR REDUCING THE INTENSITY

The clear aperture of the mirror is 59.5 inches (151.1 cm), but the entire area is not available on account of the Newtonian flat and its mounting. In order that the boundary of the shadow

[1] *Harvard Circular*, Nos. 150, 170.

[2] Kapteyn, *Plan of Selected Areas*, p. 27, 1906.

[3] Schwarzschild, *Astronomische Nachrichten*, **183**, 297, 1910.

may be regular, a central stop 23 inches (58.4 cm) in diameter is placed over the end of the tube. The free area is therefore an annulus with diameters of 59.5 and 23.0 inches.

The means used for the reduction of the intensity are mainly circular diaphragms and wire gauze screens. Trial plates were also made with a sector-shaped diaphragm having four 15° openings, with a screen of wire gauze placed in the cone of rays 30 cm from the focus, and with a 60° rotating sector mounted close to the plate. The sector diaphragm was used in studying the diffraction effect because of its radically different pattern as compared with circular apertures. The rotating sector was intended to afford further control of the diffraction effect, as the momentary and very minute disturbance of the full-aperture pattern by the edges of the sector is negligible. But the intermittent exposure introduces doubt. Schwarzschild[1] has shown that the cumulative photographic effect of a series of exposures is not the same as that produced by a single equivalent exposure. The question, however, is this: Is the photographic effect of n exposures of duration kt/n to an intensity I the same as that produced by a single exposure t to an intensity kI? If so, the reduction produced by the sector is equal to the ratio of the sum of the intermittent exposures to the total exposure, and this in turn to the angular aperture of the sector divided by 360°. A law strictly analogous to that of Talbot for visual observations would then hold. The question has been investigated by Weber at Munich, who finds that apparently such a law does hold, at least within certain limits.

Three scales thus established are in agreement with those found with the diaphragms. We may interpret the results in two ways: we may assume the validity of Talbot's law and conclude that the diffraction effect (sensibly zero for the rotating sector) is negligible for the diaphragms; or we may regard the negligibility of the diffraction effect as established by independent methods, and consider the results a confirmation of Talbot's law in its special application.

Table I gives a list of the various devices used for reducing the

[1] *Photographische Korrespondenz*, **36**, 171, 1899; *Astrophysical Journal*, **11**, 92, 1910.

intensity. Owing to the 23-inch central stop, which is always in position during photometric observations, the aperture with the 40- and 32-inch diaphragms are annuli, as is the case with the full aperture.

TABLE I

LIST OF DIAPHRAGMS, SCREENS, ETC.

Designation	Description	Reduction Constant
		Magnitudes
40	40-inch circular diaphragm	1.12
32	32-inch circular diaphragm	1.96
14	14-inch circular diaphragm	2.97
9	9-inch circular diaphragm	3.93
8	8-inch circular diaphragm	4.16
6	6-inch circular diaphragm	4.81
G	60-inch single thickness wire gauze screen	3.06
G_2	60-inch double thickness wire gauze screen	6.12
G'	14-inch single thickness wire gauze screen	3.10
G''	14-inch single thickness wire gauze screen	3.08
g	Small wire gauze screen 30 cm in front of plate	3.06
S	60° sector diaphragm, 60 inches in diameter	1.95
RS	60° rotating sector mounted in front of plate	1.95

The reduction constants in the last column were calculated for the diaphragms from their areas relative to that of the full aperture. The methods used for determining the absorption of the screens are described in the following section. The 6-, 9-, and 14-inch diaphragms are necessarily placed to one side of the central stop. They are shifted from quadrant to quadrant between the exposures in order that different parts of the mirror may be used. The threads of the two thicknesses of G_2 make an angle of 45° with each other. G' and G'' are small screens used, either singly or together, with the 14-, 9-, and 6-inch diaphragms. The resulting reduction constants are approximately 6, 7, and 8 mags., or 9, 10, and 11 mags., according as one or both of the screens are employed. The corresponding designations are $14+G'$, $14+G''$, etc. Other combinations are also possible, for example, $40+G$, $32+G$, etc., which have approximately the same reduction constants as the 9- and 6-inch diaphragms. The possibility of obtaining the same constant by quite different means affords a control which has been useful in investigating the diffraction effect and is of importance in actual observations.

4. ABSORPTION CONSTANTS OF THE SCREENS

The screens are of bronze wire 0.2 mm in diameter, and have openings 0.16×0.28 mm. Although the material was the most uniform that could be found in the market, it was clear that something more than mere calculation would be required for the determination of the absorption. A laboratory investigation was accordingly undertaken with the results summarized below. As the large screens G and G_2 were in use on Mount Wilson, the investigation was based upon a number of pieces of the material from which they were made.

Ten of these, I, II, ... X, averaging 55×65 mm, came from the edges of the original strip. Three others, about 50 cm in diameter, were cut from the centers of the large screens in order that they might clear the mounting of the Newtonian flat when attached to the end of the tube. In each of these, three areas, A, B, C; D, E, F; G, H, I, each 12.5 cm in diameter, were investigated. The first two groups relate to the 14-inch screens G' and G'', which were subsequently constructed from two of the central pieces. Finally, J, a 12.5-cm area from the edge, was added, making 20 in all that were examined.

A screen of wire gauze acts as a pair of crossed gratings. When placed in front of an objective, the transmitted light is distributed over a complicated diffraction pattern, consisting of a central diffraction disk and a large number of spectra. For light emitted by a point source, the details of this pattern are visible. If the source be a luminous surface, the image is the resultant of an infinite number of overlapping patterns of the type described—a luminous surface also, but less intense than the source. The former case corresponds to the conditions of observation, but the latter can also be utilized for the determination of the absorption.

Expressed in magnitudes, the theoretical absorption for surfaces is

$$A_s = -2.5 \log pp_1 , \tag{9}$$

and for point sources

$$A_p = -2.5 \log p^2 p_1^2 = 2A_s , \tag{10}$$

in which p and p_1 are the transmission coefficients of the crossed gratings. In other words,

$$p=\frac{b}{a}\,,\quad p_1=\frac{b_1}{a_1}\quad . \tag{11}$$

in which a and a_1 are the grating constants, and b and b_1 the spaces.

Equation (9) is rigorous, but (10) refers to ideal conditions not realized in practice. A_s is more easily measured in the laboratory than A_p. Were it not for the approximation of (10), A_p could be found by doubling the observed value of A_s. The deviations of observed values of A_p from those calculated by (10) have been investigated by du Bois and Rubens;[1] but their results are not immediately applicable. It was therefore decided to measure both A_s and A_p, as the deviation of the observed values from

$$A_p=2A_s\,,$$

compared with an extrapolation of the results of du Bois and Rubens, would afford an excellent control upon systematic errors.

The surface absorption of pieces I to X was measured with a Lummer-Brodhun photometer. The point absorption was determined for all the pieces with an apparatus presently to be described. As a further control, the values of A_s for pieces I and V were calculated by (9) and (11) from micrometric measures of the constants and spaces.

The photometer results for A_s are in Table II, the weighted means being given in the last line. To guard against systematic errors, different arrangements of the bench were used. Series 5 and 6 were with a bench-length of 1084 mm; Series 8 with 2030 mm. The respective means of the 10 pieces for the two sets of conditions were 1.506 and 1.497 mags. Other variations in the conditions were also introduced without, however, any sensible variation in the results.

The bench photometer was also used to determine the values of A_s for two thicknesses of the wire gauze (threads at 45°). The results are in Table III. The calculated values in the third column

[1] *Annalen der Physik*, **49**, 593, 1893.

were obtained by adding the means for I and II, III and IV, etc., in Table II.

TABLE II

VALUES OF A_s DETERMINED WITH THE LUMMER-BRODHUN PHOTOMETER

Series	I	II	III	IV	V	VI	VII	VIII	IX	X
1	1.518	1.458	......	......	......	......	......	......	......	......
2	1.526	......	......	......	......	......	......	......	......	......
3	1.540	1.394	......	......	......	......	......	......	......	......
4	1.515	1.428	......	......	......	......	......	......	......	......
5	1.527	1.482	1.591	1.518	1.393	1.572	1.518	1.500	1.527	1.482
6	1.522	1.463	1.562	1.512	1.404	1.562	1.526	1.468	1.526	1.463
7	1.528	......	......	......	......	......	......	......	......	......
8	1.532	1.422	1.564	1.530	1.374	1.559	1.537	1.460	1.526	1.465
Means	1.527	1.441	1.572	1.520	1.390	1.564	1.527	1.476	1.526	1.470

The values of A_s calculated from the micrometric measures were 1.518 (I) and 1.420 (V) mags. The observed values for the same pieces were 1.527 and 1.390 mags.

TABLE III

A_s FOR DOUBLE THICKNESS OF WIRE GAUZE

Pieces	A_s Observed	A_s Calculated	O−C
	Mags.	Mags.	Mags.
I and II	2.987	2.968	+0.019
III and IV	3.050	3.092	−0.042
V and VI	3.000	2.954	+0.046
VII and VIII	2.987	3.003	−0.016
IX and X	3.000	2.996	+0.004
Means	3.005	3.003	+0.002

The internal agreement of the results for a single piece is satisfactory, but the deviations in the means for the different pieces are large. It is evident, however, that these are the result of irregularities in the mesh of the screen. That they show so clearly is due to the size of the areas measured, which were limited by the construction of the photometer to a diameter of about 20 mm.

The arrangement used for measuring the absorption for point sources is shown in Fig. 1. L_1 and L_2 are lamps fed from the same

circuit; D_1 and D_2 are small diaphragms placed at the principal foci of the objectives O_1 and O_2, whose apertures and focal lengths are 12.5 and 396 cm, respectively; C is a Lummer-Brodhun cube; E, an eyepiece; and S_1 and S_2, diffusing surfaces. The screen to be investigated is placed in the parallel beam at S. The dimensions are such that the central diffraction image of D_1 formed upon D_2 is a surface of appreciable diameter, but entirely free from the spectra. Light from the latter is obstructed by the diaphragm D_2, so that the central spot of the cube is illuminated only by light from the diffraction disk. The outer area of the cube is filled with light from L_2, whose distance d from the diffusing surface S_2 may be varied until the field of E is of uniform intensity. Comparison of the positions of L_2 for uniform illumination with and without the screen S, and the application of the inverse square law lead at once to the value of A_p.

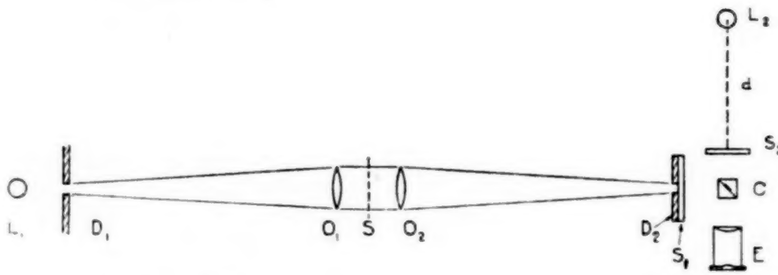

FIG. 1.—Arrangement for measurement of absorption of wire gauze screen for point sources.

The results obtained are in Table IV. For comparison, the values of $2A_s$ obtained by doubling the means in Table II are given in the fourth column.

The difference of 0.033 mag. between the means of A_p and $2A_s$ for I to X is in the direction and of the order required by the results of du Bois and Rubens. The difference between the mean A_p for I to X and that for A to J suggests a systematic difference between the edges and the middle of the strip from which the screens were constructed. The mean of the two results, namely 3.06 mags., was adopted for the 60-inch screen G, and twice this, 6.12 mags., for G_2. For G' we have the mean of A to C, 3.10 mags.; and

for G'', that of D to F, namely 3.08 mags. These are the values given in the last column of Table I along with the other reduction

TABLE IV

ABSORPTION FOR POINT SOURCES

Piece	A_p Series 1	A_p Series 2	$2A_s$ from Table II	Piece	A_p Series 3	A_p Series 4
	Mags.	Mags.	Mags.		Mags.	Mags.
I..........	3.114	3.108	3.045*	A	3.106	3.082
II..........	2.952	2.906	2.882	B	3.104	3.096
III..........	3.096	3.060	3.144	C	3.121	3.101
IV..........	2.999	2.988	3.040	D	3.121	3.102
V..........	2.960	2.916	2.810*	E	3.083	3.052
VI..........	3.206	3.181	3.128	F	3.086	3.060
VII..........	3.131	3.134	3.054	G	3.039	3.074
VIII..........	2.976	3.002	2.952	H	3.100	3.104
IX..........	3.096	3.074	3.052	I	2.961	3.002
X..........	2.922	2.933	2.940	J	3.086	3.096
Means......	3.045	3.030			3.081	3.077
	3.038		3.005		3.079	

* Mean of photometric and micrometric measures.

constants. The results for A_s serve only as a control and do not enter into the adopted constants.

5. ARRANGEMENT OF OBSERVATIONS

Photometric observations are made only during nights of uniform transparency and at least reasonably good steadiness. Bad seeing is not necessarily dangerous, unless irregular, although it increases the difficulty of measurement and prevents the registration of faint stars. Observations are discontinued when the seeing drops below 2 on a scale of 10.

The diaphragms and screens, with the exception previously noted, are attached to the end of the telescope. The practice of a symmetrical arrangement of the exposures is uniformly followed for reasons already described. For stars of intermediate brightness, the means commonly used for reducing the intensity are diaphragms of 32 and 14 inches, and the single-thickness screen G. The constant for the 32-inch diaphragm is approximately two magnitudes; that of the smaller diaphragm and the screen, three

magnitudes. For regular determinations of the scale, larger reduction constants are not practicable on account of the impossibility of properly establishing the relation between scale reading and magnitude. On the other hand, with reduction constants less than two magnitudes the scale is more seriously affected by errors in the constants. All things considered, a value of two magnitudes is a satisfactory compromise, though, as a control, it is important to include observations with other constants as well.

The order of the exposures is 60, 32, 32, 60; 60, 14, 14, 60; or 60, *G*, *G*, 60. Usually the inverse series is made upon the same plate as the direct, but occasionally, in the case of rich regions, a fresh plate is taken. A second plate is also used when the individual exposures exceed half an hour. In this case the photometric exposures on each plate are preceded and followed by short exposures of one or two minutes. The equality of the latter affords a partial test of the constancy of the atmospheric conditions. When the exposures are short, six or eight are often made on the same plate, thus: 60, 32, 14, 14, 32, 60; 60, *G*, 32, 9, 9, 32, *G*, 60; and other similar combinations. The 9-inch diaphragm included in the last arrangement is frequently useful in reaching stars near the upper limit of brightness, the main scale determination, however, being based upon the other secondary exposures.

For faint stars the arrangement, as already indicated, is very simple. It includes two exposures with the full aperture, one as long as necessary to reach the desired limiting magnitude, and the other short. The ratio of the exposure times may be taken as 10 to 1. It is desirable to have the shorter of sufficient length to register the faintest intermediate stars of known magnitude. On the other hand, the principal exposure should be such that the scale extension will not exceed two magnitudes.

To connect a very bright star with the known magnitudes of intermediate stars, greater reductions in intensity are required than can ordinarily be used. Combinations of diaphragms by means of which this may be accomplished are indicated in Section 3. The important point is that the reduced intensity of the bright star produce an image comparable in size with the full-aperture images of stars of known brightness. The determination is

strengthened by using combinations of diaphragms and screens having different reduction constants. For a fifth-magnitude star the following would be a convenient arrangement:

$$\left.\begin{array}{l}\text{Bright star: } G_2,\ 9+G',\ 6+G',\\ \text{Intermediate stars: } 60,\ 60,\\ \text{Bright star: } 6+G',\ 9+G',\ G_2,\end{array}\right\}\ \text{Exposure time, } 2^m$$

Since the approximate reduction constants are, respectively, 6, 7, and 8 mags., the apparent brightness of the reduced intensity images will be in the neighborhood of 11, 12, and 13 mags., which fall well within the limits of brightness of the intermediate stars whose magnitudes are assumed to be known. The advantage of such a program lies in the possibility of using very different means for the reduction of the intensity, and in the fact that the calculated magnitude is made to depend upon different parts of the adopted scale for the intermediate stars.

The shifting of the plate between exposures is accomplished by a screw with a graduated head, which may be moved without changing the relative position of the guiding microscope and the guiding star. One revolution displaces the plate by 1 mm, which is the shift usually employed. Except in the case of bright objects, the position of the telescope with respect to the stars remains unchanged. The distance of a given star from the axis of the mirror is therefore the same for all exposures, which simplifies the correction for distance.

The range of adjustment for the guiding microscope is such that with reduction constants of three magnitudes or less there is rarely any difficulty in finding a star bright enough for guiding. Larger constants are used only with very short exposures of two or three minutes, and if a bright star be not available, the driving of the clock is usually of sufficient uniformity.

The plates used are Seed's "Gilt Edge, No. 27." The development is with Rodinal, 1 to 25, usually for 10 minutes at 20° C. To secure uniformity and save time, a tank is used, 12 plates being developed at a time.

It is of course obvious that with an appropriate plate and filter

the methods described may be employed for the derivation of magnitudes on the visual scale (photovisual magnitudes).

6. MEASUREMENT OF THE PLATES

The images are measured with a scale placed in the common focus of the ocular and objective of a low power microscope. The plate itself is mounted upon a movable stage which can be shifted to bring any given image to the center of the field. The scale was made by exposing a plate successively to the same star, the exposure times forming a geometrical progression whose ratio was 4/3. The images are numbered from 0 to 18 in the order of decreasing size. For the law of photographic action expressed by equation (6) the magnitude difference of two successive images is

$$\Delta m = \frac{2.5}{q} \log \frac{4}{3} = \frac{0.30}{q} . \qquad (12)$$

Since Δm depends only upon the ratio of the exposure times and the quantity q, which never differs greatly from unity, the relation between scale reading and magnitude is nearly linear, with one interval corresponding to about 0.30 mag.

The measures are made by bringing a star to the center of the field and shifting the scale until the images nearest in size stand on opposite sides of the star. The reading is the number of the larger image plus the tenths of a scale interval corresponding to the difference in size of the star and the larger image. A single estimate constitutes an observation, but all plates are measured twice, usually by different observers. For the faintest images, which show little or no difference in diameter, the photographic density is the determining factor in the estimate. It is a peculiar advantage of the method that objects at the limit of visibility can be utilized, which is impossible when the images are measured micrometrically; further, the effects of poor seeing seem to be less disturbing than with the micrometric method.

The stars are identified by means of an enlarged chart of the field. In the case of rich fields, omissions and confusion are avoided by subdividing the chart into small areas by concentric circles and lines radiating from the center.

Owing to the large and irregular correction for distance from the axis of the mirror, full aperture images at distances exceeding 25 mm ($11'.3$) are not measured. The distances themselves are expressed in units and tenths of a 5-mm interval, and are read either from the identification chart or directly from the plate by means of a translucent scale of concentric circles.

7. THE DISTANCE CORRECTION

The determination of the corrections which reduce the observed scale readings to the center of the field has given more trouble than any other part of the investigation. An average correction curve has been determined, but irregular deviations occur and are responsible for a large part of the error affecting the final results. Fortunately these are accidental in character; the determination of the scale does not appear to be influenced, and the main disadvantage is a slightly lower precision in the final magnitudes.

For a reflecting telescope the theoretical aberration in the optical image of a point source is[1]

$$\gamma = \tfrac{3}{16} D'' R^2$$

in which γ is the maximum diameter of the cone of rays in seconds of arc at its intersection with the focal surface for parallel rays. R is the ratio of the aperture to the focal length, and D'' the angular distance of the object from the axis in seconds.

For $R = 1/5$ and $D'' = 680''$, the maximum distance at which full aperture images are measured, $\gamma = 5''.1 = 0.19$ mm; in the focal plane it is about 10 per cent larger still. As the diameter of the diffraction disk for a star on the axis is only $0''.075$, it is clear that the correction for distance will be large.

These figures do not, however, give the variation in the photographic image, as photographic irradiation, unsteadiness, and the distribution of intensity within the cone modify the result. Irradiation is greatest on the axis and tends to equalize the difference between axial and extra-axial images, although for bright stars the latter are always the larger. The intensity distribution is

[1] Poor, *Astrophysical Journal*, 7, 114, 1898.

such that most of the light is concentrated in a small section of the cone adjacent to the axis of the mirror. For faint stars the intensity in other parts of the cone lies below the threshold value; some of the light is thus lost, while on the axis the entire amount contributes to the photographic effect.

The dependence of the correction upon distance and brightness was determined by photographing successively, with constant exposure, the same field, the telescope being shifted slightly between the exposures. The images were measured with the photometric scale, and the readings for each star plotted against distance. The individual curves are sensibly rectilinear, but differ in slope. In accordance with the above, the inclinations for bright stars correspond to an increase in image size with increasing distance, while for faint objects the reverse is the case. For a certain intermediate brightness there is no variation, and the curves are parallel to the axis. The factor determining the slope is the size of image and depends, therefore, upon both the exposure and the brightness of the star.

To reduce the results to a useful form, the values of the correction (*D.C.*) corresponding to $D=5.0$ (unit $=5$ mm) were read from the curves for a large number of plates. The corrections were expressed in scale intervals and arranged in the order of the readings of the images to which they refer. The means of all values within certain limits of scale reading gave the relation between brightness and correction, which is also nearly linear. The results for $D=5.0$ are shown in Fig. 2. They range from $+2.5$ to -2.0 scale intervals, or from approximately $+0.75$ to -0.60 mag. For any other distance the correction may be found by an application of the linear relation for distance with the condition that for $D=0$, $D.C.=0$. In practice the values are read from a table with the distance and the scale reading as arguments and applied directly to the observed scale readings.

Although it was presumed that photographs with the 60-inch screens would require the same correction as those without screens, the matter was specially investigated for the single thickness screen *G*. The result was a curve practically identical with that of Fig. 2. The 32-inch diaphragm was similarly investigated. In this case

the corrections are so small that usually they may be disregarded. With the smaller diaphragms a different procedure showed that the corrections are sensibly zero.

The irregularities to which reference has been made could not at first be traced to a definite cause. The influence of errors in focus and of variations in the energy, temperature, and duration of development were successively studied, without finding an adequate explanation. Finally, it was noted that the deviations for different photographs of the same region were similar, and correlated with the direction as well as with the distance from the axis. This suggested temperature deformations of the mirror as the source of the difficulty. As a decisive test, the instrument was exposed to

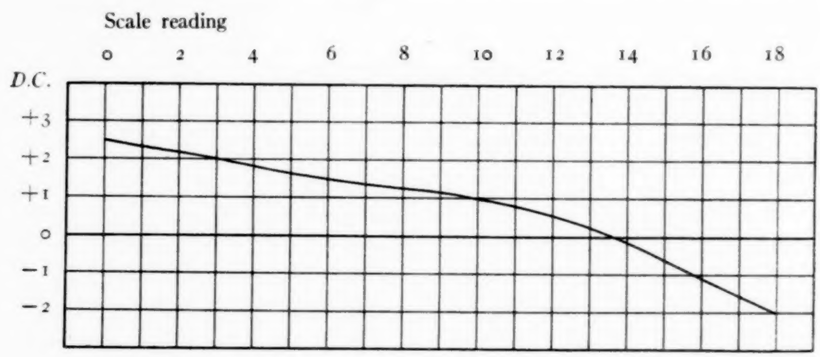

FIG. 2.—Distance correction in scale intervals for $D=5=25$ mm $=11'.3$

abnormal temperature conditions, and during the following night distance correction plates were made at intervals until normal figure had been regained. As a control upon the latter, extra-focal knife-edge photographs of the mirror were made by the method of Hartmann. The earlier distance correction plates showed the characteristic irregularities in an exaggerated form. With improvement in the figure of the mirror these decreased, and with normal figure the curve was approximately that given above. The location of the difficulty has made it possible to reduce the uncertainty; but it cannot be wholly removed, for small deformations, apparently without influence upon other kinds of observations, are appreciable in photometric work.

8. REDUCTION OF THE MEASURES

a) *Intermediate stars.*—After the measurement of the plates with the photometric scale, the mean scale reading is formed for the images of each star made with the same aperture. Thus, for the series of exposures represented by 60, 32, 32, 60, the two readings on each of the 60-inch images are combined. A similar combination gives the mean for the diaphragm images. Results requiring correction for distance are treated in the manner described in the preceding section. The difference in the corrected readings

$$s'-s=\Delta s \tag{13}$$

corresponds to the reduction constant Δm of the secondary exposure. Each of the brighter stars gives a relation of the form of (13), and the problem is to find

$$m=f(s) \tag{14}$$

subject to the condition

$$m+\Delta m=f(s+\Delta s)=f(s')\,. \tag{15}$$

An analytical solution has been given by Schwarzschild.[1] Usually Δs is a function of s, but the fact that the relation between magnitude and scale reading is nearly linear reduces the variation in Δs to a minimum. Not infrequently it may be regarded as constant. In this case, for full aperture

$$m=\alpha+\beta s \tag{16}$$

and for reduced intensity,

$$m=\alpha+\beta s'-\Delta m\,, \tag{17}$$

in which α is a constant depending on the zero point, and

$$\beta=\frac{\Delta m}{\Delta s}\,. \tag{18}$$

In any case, Δs is plotted with s as abscissa. If the curve shows Δs to be sensibly constant, the magnitudes are calculated at once by equations (16) to (18). Otherwise, the scale is calibrated by a process based upon the Schwarzschild solution. This is illustrated

[1] *Astronomische Nachrichten*, **172**, 65, 1906.

by the example in Table V, which relates to a plate with the 14-inch diaphragm and is chosen because of the unusually large variation in Δs.

TABLE V

CALIBRATION OF THE SCALE

Plate 808 P, Apertures 60 and 14, $\Delta m = 2.97$ mags.

s	Δs	s'	m_{60}		m	m_{14}
0..........	10.5	10.5	0.00		2.97	−2.97
				28		
1..........	10.5	11.5	0.28		3.25	2.69
				29		
2..........	10.4	12.4	0.57		3.54	2.40
				28		
3..........	10.3	13.3	0.85		3.82	2.12
				28		
4..........	10.2	14.2	1.13		4.10	1.84
				29		
5..........	10.0	15.0	1.42		4.39	1.55
				28		
6..........	9.7	15.7	1.70		4.67	1.27
				28		
7..........	9.3	16.3	1.98		4.95	0.99
				28		
8..........	9.0	17.0	2.26		5.23	0.71
				29		
9..........	8.6	17.6	2.55		5.52	0.42
				28		
10..........	8.2	18.2	2.83		5.80	−0.14
				28		
11..........	7.8	18.8	3.11		6.08	+0.14
				30		
12..........	....	....	3.41		....	0.44
				32		
13..........	....	....	3.73		....	0.76
				33		
14..........	....	....	4.06		....	1.09
				36		
15..........	....	....	4.42		....	1.45
				39		
16..........	....	....	4.81		....	1.84
				42		
17..........	....	....	5.23		....	2.26
				48		
18..........	....	....	5.71		....	+2.74

The process is begun by writing in the first column the scale numbers from 0 to 18. Opposite these are inserted the corresponding values of Δs read from the curve. The horizontal summation of the first two columns gives the third. Values of s and s' standing opposite each other correspond to the magnitude difference

Δm, which is the condition used for the derivation of the magnitudes in the fourth and fifth columns relating, respectively, to s and s'.

For small values of s, the Δs curve, in this case, is parallel to the axis. This implies that for s less than

$$s+\Delta s=1.0+10.5=11.5,$$

the relation of magnitude to scale reading is linear, with the proportionality factor β, by (18), equal to 0.283 mag. ($\Delta m=2.97$ mags., $\Delta s=10.5$). The products of β into the scale numbers 0 to 11 are therefore formed and inserted in the fourth column. To these are added the reduction constant 2.97, the results being entered in the fifth column. By the condition (14), the latter series of magnitudes corresponds to the scale readings in the third column, and it is now possible to find by interpolation the values of m for the integral values of s from 12 to 18. The results are entered in the fourth column, small arbitrary changes being made at two or three points in order to secure a greater regularity in successive differences.

With the data in the first and fourth columns, the magnitudes of all the stars on the plate may be obtained. For the diaphragm exposures it is necessary to subtract the reduction constant from the results of the interpolation in order that they may be referred to the zero point of the magnitudes given by the full aperture images. This is best done by subtracting once for all the constant from the tabular values of m_{60}. The results are in the last column under the heading m_{14}, and are to be used in interpolating the magnitudes for the secondary exposures. The zero point is fixed by the method used for finding the values of m_{60}. Its relation to the true zero point must be found by a comparison with stars of known magnitude.

The process described is readily adapted to such variations in the curve for Δs as have been found to occur in practice. The assumption of a linear relation between s and m can almost always be made for some part of the scale. Should this prove impossible, more general relations proposed by Schwarzschild may be employed.

In case more than one set of diaphragm exposures is made upon

the same plate, as for example with the arrangement 60, 32, 14, 14, 32, 60, two separate determinations of the scale are made, using the combinations 60–32 and 60–14. When the 9-inch diaphragm is also added, there are not ordinarily sufficient data to permit a separate determination; but the images thus secured are easily reduced by means of the scales established with the other diaphragms, and strengthen the results for the brighter stars which are always somewhat uncertain.

b) *Faint stars.*—The images of the two exposures are measured, corrected for distance error, and plotted with the known magnitudes as ordinates. The ordinates of the two curves thus obtained differ by Δm. Were the law of photographic action expressed by equation (6) rigorously true Δm would be constant. We have already seen, however, that the value of Δm decreases with decreasing intensity, and experience shows that its change is slow and nearly linear. Its values are read from the sheet and extrapolated over the intensity interval of the faint stars registered by the long exposure. The addition of the extrapolated values to the ordinates of the short-exposure curve in the region of faint intensity gives a series of points through which the long-exposure curve may be produced, thus establishing the relation between magnitude and scale reading for the faint stars. The magnitudes of the latter are then read directly from the extended curve.

The following series of values from two plates indicate the character of the change in Δm:

Scale Reading	Pl. 1332 P	Pl. 1360 P
2	....	2.22
4	2.35	2.17
6	2.20	2.11
8	2.12	2.07
10	2.03	2.03
12	1.92	1.98
14	1.80	1.90

With this degree of uniformity, it is probable that the extrapolation over 5 or 6 scale intervals can be made with some accuracy, and the agreement of the results from different plates seems to justify this opinion.

c) ***Bright stars.***—The process to be employed for the reduction is sufficiently indicated by the account of the method already given. Details and illustrative results may be found in "The Photographic Magnitude Scale of the North Polar Sequence."[1]

9. THE DIFFRACTION EFFECT

The relatively unimportant influence of a change in the diffraction pattern was established in two ways: first, by determining the increase in the diameter of star images produced by small errors in focus, and, second, by examining the accordance of the magnitude scales for the same group of stars established by the different methods of reducing the intensity.

a) ***Errors of focus.***—When the aperture is decreased by a circular diaphragm, the diameter of the diffraction disk is increased in the inverse ratio of the apertures. The intensity is, at the same time, reduced by the equivalent of Δm magnitudes. Will the reduced intensity of a star S, distributed over the enlarged diffraction disk, produce the same photographic effect as the equivalent intensity of a star Δm fainter than S photographed with the full aperture? It is a question as to the equality of photographic effects when equal quantities of light act upon areas of the sensitive film differing in size. The matter may be studied by measuring the diameters of focal and slightly extra-focal images of the same star photographed with the full aperture and constant exposure.

Two series of photographs, 18 plates in all, were made with different focal settings, the true focus being determined with the knife-edge. Five or six two-minute exposures were made on each plate, and upon each, 15 stars, on the average, were measured with the photometric scale. The mean results for an image of average brightness are in Table VI. The first column contains the deviations from the true focus, and the second and third, the corresponding change in the image expressed in magnitudes. In forming the means in the fourth column, the second series was given double weight. The quantities in the last column are the means for equal positive and negative focal deviations.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 70; *Astrophysical Journal*, **38**, 241, 1913.

With the full aperture, the diameter of the cone of rays 0.005 inch from the focus is very nearly ten times that of the diffraction disk. The circular aperture giving a diffraction disk of this diameter is one-tenth the full aperture, or 6 inches. From Table VI it appears that the diffraction effect for such a diaphragm should be of the order of 0.03 mag. As the corresponding reduction constant is nearly five magnitudes, the resulting error in the scale must be very small.

TABLE VI

RELATION BETWEEN FOCUS AND SIZE OF IMAGE

ΔF (Inches)	Δm			Mean Ex-F. and In-F.
	Series I	Series II	Mean	
+0.015	−0.19	....	−0.19	−0.15
+0.010	−0.19	−0.06	−0.10	−0.08
+0.005	−0.08	0.00	−0.03	−0.03
0.000	0.00	0.00	0.00	0.00
−0.005	0.00	−0.04	−0.03	....
−0.010	−0.04	−0.09	−0.07	....
−0.015	−0.09	−0.14	−0.12	....

Strictly speaking, the matter should have been investigated separately for stars of different brightness, but the material available, when thus divided, is insufficient for a precise determination of the minute quantities involved.

b) *Accordance of magnitude scales.*—The best test of the diffraction effect is afforded by a comparison of results found with the wire gauze screens and the diaphragms. With screens the central diffraction disk is unchanged, and the results are free from the error in question. Assuming the screen constant to be known with precision, the comparison indicates at once the errors to be expected in using the diaphragms.

A series of photographs was made specially for an investigation of the question shortly after photometric work with the reflector was begun. At that time the importance of a rigid temperature control of the instrument was not realized and, as a consequence, difficulties arose in the correction of the observations for distance. Two separate reductions were made by entirely different methods. In the first, the distance error entered as an unknown. In the

second, it was found by comparing the full aperture images with the results of a preliminary reduction based upon exposures made wholly with diaphragms of 32, 14, 8, and 6 inches, for all of which the field of view is sensibly normal.

Although the mean scales from the two reductions differ considerably, the deviations of the results of the different methods of intensity reduction from the two means are small. The former difference arises from the uncertainty in the distance correction, and is of no great importance for the moment. The deviations from the separate means represent the systematic differences between the different methods for establishing the scale. These are shown in Tables VII and VIII, the unit being 0.01 mag.

TABLE VII

Systematic Differences—First Reduction

Mean Mag.	60–*G*	60–32	60–14	60–8,60–6	32–8,32–6	No. Stars
0.61	−5	+8	−7	+ 8	+ 1	5
1.55	+6	+7	−2	− 5	−11	5
2.44	−1	+1	+6	+ 2	−12	10
3.58	+2	+1	+5	−13	+ 2	8
4.24	0	+1	−1	+ 2	− 2	9
4.74	−3	−2	0	− 2	+9	15
5.21	−1	0	−2	+ 4	....	12
5.53	+2	−3	0	0	....	12
5.98	+8	−4	−4	+ 2	....	7
No. scale determinations	10	10	11	7	6	83 44

TABLE VIII

Systematic Differences—Second Reduction

Mean Mag.	60–G	60–32	60–14	60–8 60–6	32–14	32–8 32–6	60–*S*	60–*RS*	No. Stars	Average Deviation
0.38	+7	−6	−5	+1	+ 3	+1	+6	−3	9	±13
1.77	+4	−3	+5	−1	+ 6	−8	−4	−3	10	8
2.95	+2	+1	+1	−9	− 1	+1	−2	+1	10	10
3.77	+3	0	+3	+3	−11	−5	−6	−3	10	10
4.11	−2	+1	0	−2	+ 3	+9	−5	−6	10	11
4.45	−4	+3	−1	+3	......	......	−1	0	10	14
4.72	−3	+2	−1	+1	......	......	+3	+3	10	11
5.13	−4	+2	−4	+3	......	......	+9	+9	10	±15
No. scale determinations	10	10	11	7	5	7	2	3	79 55	±11^5 ± 0.097 mag. (P.E.)

The zero points of the mean magnitudes given in the first columns are arbitrary. The interval actually covered was from 9.5 or 10 to about 15. The column headings indicate the apertures used for establishing the scale whose deviations are given in the body of the table. These are generally small, and in only a few cases are they progressive. Even here the result is apparently due quite as much to the uncertainty in the distance correction as to other sources of systematic error, for it will be noted that the divergences for the 60–32 combination of apertures are in opposite directions in the two reductions.

Although the results for the sector diaphragm (*S*) and the rotating sector (*RS*) are of low weight, it is of interest to note that their deviations are not appreciably greater than the others. The diffraction pattern for the former is very different from that of a circular or annular aperture, and that there is no marked divergence may be regarded as significant. The interpretation of the results for the rotating sector has already been mentioned.

As a further test of the accordance of scales established by different methods of reducing the intensity, results from an unpublished investigation of the Polar Sequence are given in Table IX. In this instance the distance correction used was that found by the method of Section 7. The magnitudes were calculated by the interpolation process given in Section 8. The mean scale, at least as far as 15.5, is probably close to the true value, as the differences between the Mount Wilson and Harvard results shown in the second column are small.

As before, small progressive deviations are in evidence; but these are perhaps caused by errors peculiar to the plates, rather than by systematic differences inherent in the methods of establishing the scale. If, for example, all the deviations in Tables VII–IX relating to combinations of the full aperture with diaphragms of 32, 14, 8, and 6 inches be formed into one set of means, and those derived with 60–*G* and 60–*g* into another, we find the results of Table X, in which there remains little trace of progressive divergence.

The results in Tables VII–X relate to stars of intermediate brightness. For the internal agreement of scales found for the

brighter stars, reference may be made to "The Photographic Scale of the North Polar Sequence."[1] The mean scale for these objects is in less satisfactory agreement with that of the Harvard Observatory, but there is no evidence of appreciable systematic difference between the results found by the different methods of reducing the intensity.

TABLE IX

SYSTEMATIC DIFFERENCES—POLAR SEQUENCE STARS

Mean Mag.	Mt. W. −H.*	60–*G*	60–*g*	60–32	60–14	60–8	60–*S*	No. Stars	Average Deviation
10.78..........	− 2	−12	− 6	+ 3	−2	+18	−12	7	±16
12.27..........	− 4	0	− 7	+ 3	−3	− 6	− 4	7	11
13.21..........	+ 3	− 2	− 5	− 1	+7	+10	− 1	10	13
13.91..........	0	+ 3	+ 5	− 3	+5	− 8	+14	11	14
14.44..........	+ 4	+ 6	+ 4	− 2	+1	+ 4	+ 2	11	12
15.05..........	− 5	+12	+ 5	− 4	−4	−10	+ 9	11	12
16.07..........	−21	+19	+ 9	− 6	−4	−21	+25	5	20
16.79..........	−35	+14	+14	−11	+1	−16	......	5	±13
No. scale determinations.....	......	5	2	13	4	2	3	67 29	±13.0 ± 0.110 mag.(P.E.)

* Since the publication in *Contribution* No. 70 of a similar comparison with the Harvard results, the calculations have been revised and small corrections to the zero points of the separate scales have been applied. This and a different grouping of the stars accounts for the difference between the above results and those originally given. The number of stars in the next to the last column does not apply to the differences Mt.W.−H.

TABLE X

MEAN SYSTEMATIC DIFFERENCES BETWEEN DIAPHRAGMS AND SCREENS

Mean Mag.	Diaphragms	Screens	*D−S*	Mean Mag.	Diaphragms	Screens	*D−S*
0.52	+2	−4	+6	4.27 ...	+1	0	+1
1.80	+1	+2	−1	4.77 ...	0	0	0
2.80	+2	−2	+4	5.17 ...	0	+1	−1
3.65	0	+2	−2	5.78 ...	−2	+5	−7

In all of the foregoing comparisons it is, of course, to be understood that the errors of the reduction constants, as well as a possible diffraction effect, are included in the residuals. It seems to be reasonably clear that neither of these can be of any great importance.

As far as the diffraction effect is concerned, the result is perhaps what might have been anticipated. If we consider the ideal case

[1] *Loc. cit.*

in which the light is assumed to act only upon a region of the plate of the same size as the diffraction disk, we shall in all cases be dealing with areas smaller than the smallest star image. In other words, the greater part of the image is the result of photographic irradiation or other causes. This being the case, the actual size of the diffraction disk, within the limits considered, would presumably be a matter of minor importance. Actually, however, the area of the plate affected is larger than the diffraction disk, because of the oscillations of the image produced by atmospheric inequalities, and this tends to minimize the differences which might otherwise exist between large and small apertures.

10. PRECISION OF THE RESULTS

The time has scarcely arrived for a final estimate of the precision of results derived by the methods described. Nevertheless, as an indication, the quantities in the last columns of Tables VIII and IX are given. In Table IX these are the average deviations of a single magnitude, based upon the mean of two images on the same plate, from the mean established by 29 separate determinations of the scale. Fifteen plates were available with a possible maximum of 30 scale determinations, but the rejection of one set of diaphragm exposures reduced the number to 29. Over 1500 magnitudes are involved, and of this number, none were excluded in forming the means. There is some tendency for the deviation to increase as the ends of the scale are approached. The average for the entire series is 0.130 mag., corresponding to a probable error of 0.110 mag. The result includes not only the accidental errors, but also the systematic deviations of the individual scales from the mean for a range of six magnitudes.

To obtain an estimate of the latter, the differences between the magnitudes derived for each star from the full- and reduced-intensity exposures on each plate were formed whenever possible. These differences are free from scale-error, and their mean value without regard to sign is a measure of the remaining errors. From 400 such differences for the Polar Sequence stars a mean of 0.142 mag. was found. The probable error in a single magnitude, exclusive of scale error, is therefore 0.085 mag. The difference

between this and the entire probable error of 0.110 mag. is the part contributed by errors of scale. The probable error corresponding to the latter is 0.070 mag. This result is not unsatisfactory when the scale interval covered is considered, but the remaining error represented by 0.085 mag. is large. As already pointed out, much of this is doubtless due to temperature deformations of the mirror; the accordance of the scale readings is such that the error of estimate, which besides distance error is the only source of any considerable uncertainty, must contribute something less than one-half of the outstanding 0.085 mag.

Similar results follow from Table VIII. Here, however, the deviations were calculated for the mean of the magnitudes found with full and reduced intensity whenever the secondary image was registered, which occurs for about one-third of the cases. Taking this into account, the probable errors from Tables VIII and IX are in good agreement.

The Polar Sequence plates, as well as those for the study of the diffraction effect, were made under unfavorable conditions of temperature. Latterly the instrument has been as carefully protected as possible, and the results are more precise. As an illustration, there may be noted the probable errors, including scale divergence, for each of six selected areas. These are 0.082, 0.079, 0.069, 0.090, 0.109, and 0.111 mag., respectively. The last two are the same as that found above, but the others are smaller, and the mean of 0.085 mag. shows an appreciable gain in precision.

Each of these values is based upon an average of 300 magnitudes. Four plates were used for each area, and the deviations are referred to the mean of six or more determinations of the scale.

No detailed examination has yet been made of the relative precision afforded by the different methods of reducing the intensity. It seems likely, however, that there is not much preference in this respect between the different circular diaphragms and the screens, at least for reduction constants not exceeding three magnitudes. The smaller number of values of Δs obtained with large reduction constants is compensated for by the size of the constant itself. But beyond the limit mentioned, there is difficulty in calibrating the

photometric scale. The sector diaphragm is not to be recommended, for although accordant scales have been established by its use, it affects the definition, and the accidental errors are large. The rotating sector is probably not to be trusted without assurance that for the special conditions involved the equivalent of Talbot's law may be applied.

MOUNT WILSON SOLAR OBSERVATORY
November 25, 1913

THE RADIAL VELOCITIES OF ONE HUNDRED STARS WITH MEASURED PARALLAXES[1]

By WALTER S. ADAMS AND ARNOLD KOHLSCHÜTTER

A part of the observing program of the 60-inch reflector for radial velocity determinations during the past three years has consisted of stars fainter than magnitude 5.5 on the visual scale for which observations of parallax are available. The measurement of the radial velocity completes the list of fundamental constants for such stars, and makes it possible to calculate their real motions in space. This fact is of especial interest since these stars are in general those nearest to the solar system, and may be expected, in view of their large proper motions, to show high average radial velocities. A knowledge of the spectral types of these stars is also of much importance, since their absolute magnitudes may be calculated with the aid of the parallax determinations.

The instrument employed for all of the observations has been the Cassegrain spectrograph adapted for use with one prism. Two cameras have been used: first, a lens with a focal length of 102 cm for stars between magnitudes 5.5 and 6.5; and second, a lens of 46 cm focal length for stars fainter than 6.5. The first is a special triplet made by Brashear, and the second a Cooke lens of the "Astrophotographic" type. Both lenses give excellent definition, but the field of the second lens is considerably the wider. As a result the measures which, in the case of the larger scale photographs, are limited to the region between H_γ and H_β, are extended for the smaller scale plates to the region between λ 4200 and H_β. The approximate linear scale of the photographs at H_γ is as follows:

102-cm camera............	1 mm = 16 angstroms
46-cm camera............	1 mm = 36 angstroms

The exposure times with the two cameras are nearly in the ratio of 5 to 1, allowing for a well-measurable width of spectrum in

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 79.

both cases. To illustrate the efficiency of the large reflector and spectrograph under good conditions of transparency and definition the following list of data for a few photographs obtained with the short camera is taken from the observing book. The definition is on a scale of 5.

TABLE I

Star	Mag.	Spectrum	Exposure Time	Slit-Width	Definition
			Min.	mm.	
Lal. 27742	6.8	G2	27	0.038	3
W.B. 15^h720	6.8	G9	30	0.038	3
Groom. 1855	7.4	G8	61	0.038	3-4
OΣ 298	7.9	K0	75	0.038	3-4
Lal. 5761	8.0	A3p	77	0.038	3
Groom. 34	8.2	Ma	122	0.038	2-3
Lal. 21185	7.6	Ma	92	0.033	4

In the course of measurement we have found it desirable to select somewhat different stellar lines on the photographs taken with the two cameras. On the larger scale plates most of the lines used are of moderate intensity, and relatively few of them are direct comparisons with the iron lines of the comparison spectrum. With the small scale given by the short camera, however, the strong lines become so narrow that they are well measurable, and a considerable proportion of the lines used are direct comparisons with the stronger arc lines such as λ 4260, λ 4383, λ 4404, etc. Our experience has shown that in spectra of types F to K5 the use of such strong lines avoids many of the difficulties arising from uncertainties in the wave-lengths of blended lines. In stars of types K5 to Mb, on the other hand, the relative intensities of the stellar lines as compared with the sun are so greatly modified that even the strong iron lines may be affected by neighboring lines which blend with them, and much care has to be exercised in the selection of those to be used. The number of lines measured is usually from ten to fifteen.

In the tabulation of our results for the radial velocities of 100 of these stars we have adopted the plan of giving the values for the individual plates rounded off to the nearest kilometer. The mean

values for the long camera photographs are given to fractional parts of a kilometer, while for the small scale plates they are left in even kilometers. The fractional parts, however, are used in the formation of the mean values in both cases so that no error enters into the mean from this source. With the small linear scale of the spectrograms taken with the short camera it is of course evident that a fairly large range is to be expected among the separate determinations of radial velocity. The occasional discrepancies found among the long camera results are due in part to the character of some of the stellar spectra, and in part may represent actual variations of velocity. We do not, however, consider such variations as established from these results.

The spectral classifications given in Table II are made from our own photographs, the data already published for these stars being somewhat scanty. Occasionally large differences are found between our values and such as are given by the Harvard observers, and in these cases we have taken especial care to check the results. The Harvard system of classification is used throughout.

In the accompanying table (II) the right ascension and declination given for each star are for the epoch 1910. The magnitudes are taken from *Publications of the Astronomical Laboratory at Groningen*, No. 24, and are on the Harvard visual scale. The column v is the mean observed radial velocity, and v' is this velocity corrected for the sun's motion, assuming the apex at $\alpha=270°$, $\delta=+30°$, and a solar motion of 20 km. The quantities μ, ψ, and π denote as usual the star's proper motion, the position angle of the proper motion, and the absolute parallax, and are taken from the Groningen publication already referred to. The absolute magnitudes given in the last column are also taken from this publication. The remaining three columns of the table denoted by A, Δ, and V give the right ascension and declination of the star's apex and its absolute motion in space. These quantities were computed with the aid of the convenient formulae derived by Dr. Wilson and published in *Lick Observatory Bulletin*, No. 214, where the values for a large number of the brighter stars with great proper motions are given. The formulae were adapted for logarithmic computation, and the

TABLE II

Radial Velocities of Stars with Measured Parallaxes

Star	R.A. 1910	Dec. 1910	Mag.	Spec.	v from Separate Plates	Mean v	v'	μ	ψ	π	A	Δ	V	Abs. Mag.
					km	km	km						km	
Pi. 23^{h}267	0^h 0^m.1	+34°10′	6.2	G0	+ 6 + 4 + 2	+ 3.9	+ 9.5	0″.77	84°	+0″.01	....	....	....	....
Brad. 3212	0 1.9	+28 32	6.2	K0	− 8 − 6 − 9	− 7.7	− 3.0	0.43	116	+0.15	232°	+10°	7	7.1
Groom. 34	0 13.0	+43 30	8.2	Ma	+ 4 + 1 + 3	+ 3	+ 9	2.85	82	+0.28	100	+29	35	10.4
26 *Androm*	0 13.9	+43 17	6.0	B9	+ 9 + 7 + 6	+ 7.1	+ 13.2	0.04	63	−0.02	....	....	....	....
Pi. 0^{h}137	0 34.5	+ 2 37	7.4	G3	− 28 − 29 − 30	− 29	− 31	0.79	68	+0.02	111	+26	179	....
Lal. 1045	0 35.8	+39 43	7.5	K1	− 60 − 65 − 61	− 62	− 58	0.81	152	+0.02	50	−53	185	....
Lal. 1299	0 43.6	+ 4 49	5.8	K2	− 12 − 11 − 12	− 11.8	− 14.2	1.34	148	+0.16	172	−62	28	6.9
Lal. 1966	1 4.1	+61 1	7.9	F3	−326 −325 −323	−325	−319	0.69	90	+0.08	188	−59	320	7.4
Fed. 263	1 34.8	+66 28	7.6	G4	+ 19 + 13 + 15	+ 15	+ 21	0.75	108	+0.04	94	+10	74	5.4
Pi. 1^{h}159	1 41.2	+63 25	5.7	K1	+ 5 + 3 − 2	+ 1.9	+ 7.5	0.62	113	+0.06	96	+ 6	31	4.7
Lal. 4141	2 10.2	+23 53	6.9	G2	+ 3 − 50 − 3 − 1	− 1	− 6	0.60	106	+0.03	123	−11	78	....
Pi. 2^{h}123	2 31.1	+ 6 28	5.9	K2	+ 22 + 25 + 23 + 20	+ 22.5	+ 13.1	2.31	51	+0.14	120	+52	78	6.7
Lal. 4855	2 33.3	+30 28	7.2	G0	− 95 −103 − 99	− 99	−103	0.60	228	+0.02	287	−44	177	....
Lal. 5490-6	2 56.7	+61 23	6.7	G1	− 9 − 7 − 6	− 7	− 4	1.00	134	+0.06	98	−23	60	5.4
Lal. 5761	3 3.1	+26 0	8.0	A3p	−143 −144 −144	−144	−151	0.90	192	+0.04	245	−55	179	6.0
Groom. 864	4 35.2	+41 58	7.3	G0	+101 +104 +109	+105	+100	0.73	127	+0.03	107	+12	141	....
Groom. 884	4 45.1	+45 42	7.1	F9	+ 24 + 26 + 27	+ 26	+ 22	0.68	145	+0.09	104	+17	29	6.9
Lal. 9091	4 46.2	+10 55	7.0	F5	+ 35 + 41 + 39	+ 38	+ 24	0.10	107	−0.02	....	....	....	....
Lal. 9109	4 46.8	+13 30	6.7	F8	+ 35 + 36 + 43	+ 38	+ 24	0.13	117	+0.01	....	....	....	....
Lal. 11196	5 50.9	+13 56	6.5	G1	− 4 + 1 − 2	− 2	− 16	0.69	137	+0.06	196	−38	49	5.3
W.B. 6^{h}1500	6 51.8	+ 1 17	7.7	G4	− 11 − 12 − 14	− 12	− 29	0.57	170	+0.01	....	....	....	....
Pi. 6^{h}305	6 57.8	+29 30	6.0	F9	+ 22 + 20 + 19	+ 20.3	+ 10.6	0.82	168	+0.07	134	−40	42	5.1
Lal. 13849	7 4.8	+21 24	6.5	G7	− 11 − 17 − 15	− 14	− 26	0.52	197	+0.11	291	−35	27	6.6
Groom. 1281	7 9.2	+47 24	5.6	F9	+ 89 + 89 + 86	+ 88.0	+ 84.2	0.18	171	+0.07	107	+46	83	4.7
Lal. 15290	7 47.8	+30 54	8.2	F7	−247 −241 −239	−242	−250	1.97	158	+0.04	82	+62	316	6.5
Lal. 15565	7 54.9	+29 29	6.9	G4	+ 15 + 13 + 13	+ 14	+ 6	1.19	187	+0.09	117	−55	46	6.7
Pi. 7^{h}321	8 6.1	+32 45	7.0	G2	+ 26 + 30 + 25	+ 27	+ 20	0.82	213	+0.05	84	−29	62	5.4
Lal. 16304	8 14.1	−12 20	6.0	G9	+ 31 + 33 + 28	+ 31	+ 15	1.03	167	+0.13	55	−59	38	6.6
32 *Lyncis*	8 27.5	+36 44	6.1	F2	+ 2 + 2 + 1	+ 1.7	− 3.4	0.14	274	+0.09	294	+40	18	5.9
55ρ^1 *Cancri*	8 47.2	+28 41	6.1	K1	+ 30 + 26 + 28 + 27	+ 27.6	+ 21.1	0.53	243	+0.08	90	+24	27	5.7

Lal. 18115 *Pr.*	9 8.3	+53 5	7.9	K8	+ 8 + 11 + 13	+ 11	+ 12	1.69	248	+0.16	63	+12	36	9.0
Lal. 18115 *Fol.*	9 8.3	+53 5	7.9	K8	+ 11 + 13 + 9 + 8	+ 10	+ 11	1.69	248	+0.16	62	+11	36	9.0
Lal. 18286	9 12.6	+28 58	7.3	G9	− 16 − 11 − 20	− 16	− 21	0.52	172	+0.01	.....	.....	.....	.....
11 *Leo Min.*	9 30.3	+36 13	5.5	G7	+ 13 + 9 + 14	+ 12.2	+ 9.6	0.74	250	+0.04	66	− 6	72	3.4
20 *Leo Min.*	9 55.9	+32 23	5.6	G1	+ 55 + 57 + 56	+ 55.9	+ 53.7	0.68	230	+0.07	128	+15	60	4.9
Groom. 1618	10 5.1	+49 55	6.8	K7	− 32 − 30 − 28	− 30	− 28	1.45	249	+0.18	279	−38	33	8.1
39 *Leonis*	10 12.4	+23 34	5.8	F5	+ 38 + 39 + 39 + 40	+ 38.8	+ 35.6	0.44	258	+0.09	139	+34	38	5.7
Brad. 1433	10 16.9	+41 41	5.9	F7	− 7 − 9 − 6 − 6 − 9	− 7.3	− 6.3	0.19	224	+0.11	328	+ 6	9	6.1
Groom. 1646	10 22.5	+49 18	6.5	F9	− 11 − 7 − 9 − 5	− 8	− 5	0.90	173	+0.08	194	−41	47	6.1
Pi. 10^h96	10 28.3	+49 39	7.6	F7	− 24 − 28 − 27	− 26	− 23	0.30	60	+0.04	285	+ 2	60	5.8
Groom. 1697	10 47.5	+70 20	6.1	K0	+ 19 + 14 + 18	+ 17.1	+ 24.7	0.42	260	−0.01	.....	.....	.....	.....
Lal. 21185	10 58.4	+36 35	7.6	Ma	− 91 − 84 − 85	− 87	− 85	4.77	186	+0.40	330	−66	96	0.6
51 *Leo Min.*	11 0.4	+25 42	7.5	G0	− 10 − 8 − 7	− 9	− 9	0.45	260	+0.04	63	− 3	37	5.4
Groom. 1757	11 11.6	+49 58	6.0	K0	− 3 0 − 2 + 1	− 1	+ 4	0.09	256	−0.08	.....	.....	.....	.....
Groom. 1774	11 19.0	+37 43	6.9	A0	− 12 − 14 − 12	− 13	− 9	0.06	259	+0.03	317	+ 5	14	4.3
Groom. 1812	11 34.0	+45 37	6.3	F9	− 19 − 15 − 19	− 17.6	− 11.8	0.63	273	+0.04	67	+ 1	60	4.2
Pi. 11^h218	11 57.9	+43 37	6.8	G4	− 13 − 15 − 11	− 13	−	0.67	213	−0.03	.....	.....	.....	.....
Groom. 1855	12 5.1	+40 45	7.4	G8	− 3 − 3 − 3	− 3	+ 4	0.34	260	+0.06	103	+29	8	6.4
33 *Virginis*	12 41.8	+10 3	5.8	K1	+ 51 + 51 + 51	+ 51.0	+ 55.8	0.54	148	−0.11	.....	.....	.....	.....
Lal. 24414–6	13 4.3	+ 5 43	6.9	G3	+ 21 + 25 + 21	+ 22	+ 28	0.72	175	+0.01	.....	.....	.....	.....
Lal. 25224	13 35.2	+11 12	5.5	A4	− 25 − 23 − 22	− 23.4	− 14.6	0.08	300	+0.26	341	+16	22	7.6
Lal. 26289	14 18.6	+ 1 40	6.3	G0	− 14 − 21 − 16 − 19	− 17.8	− 7.7	0.55	156	−0.01	.....	.....	.....	.....
Pi. 14^h212	14 52.2	−21 1	5.8	K6	+ 20 + 20 + 21	+ 20.1	+ 27.5	2.07	150	+0.17	296	−44	62	6.9
Lal. 27742 *Br.*	15 8.8	+19 38	6.8	G2	− 34 − 33 − 39	− 35	− 20	0.68	296	0.00	.....	.....	.....	.....
Lal. 27744	15 9.3	− 1 0	6.7	G8	− 72 − 71 − 67 − 70	− 70	− 58	1.38	247	+0.13	79	− 7	67	7.3
6 *Serpentis*	15 16.5	+ 1 2	5.5	K4	+ 11 + 10 + 11	+ 10.5	+ 23.8	0.12	209	+0.13	250	+14	27	6.1
Lal. 28358	15 26.8	+57 45	6.9	F3	− 31 − 34 − 32	− 32	− 16	0.29	301	+0.03	101	− 8	38	.....
OΣ 298	15 32.8	+40 8	7.9	K0	− 71 − 73 − 70	− 72	− 55	0.48	279	+0.05	90	−28	65	6.2
W.B. 15^h720	15 32.9	+40 6	6.8	G9	− 66 − 75 − 68 − 73	− 71	− 54	0.48	279	+0.05	91	−28	64	5.1
Lal. 28607	15 38.3	−10 39	7.3	A2p	−166 −168 −172 −175	−170	−158	1.17	253	+0.03	100	− 3	233	.....
Groom. 2305	16 1.8	+39 24	6.8	G5	− 60 − 60 − 59	− 60	− 42	0.56	275	+0.07	103	−29	50	6.1
Lal. 29437–8	16 4.8	+ 6 39	6.0	K1	− 1 − 5 − 1 − 2	− 2.3	+ 13.9	0.79	162	+0.01	.....	.....	.....	.....
Lal. 30024–6	16 25.0	+18 36	7.0	G7	− 36 − 39 − 34	− 36	− 18	0.49	322	+0.09	85	+33	32	6.8
Lal. 30044–5	16 26.1	+ 4 25	7.3	F6	− 49 − 54 − 54	− 52	− 35	1.46	198	+0.01	(161	−72	683)	.....
Lal. 30699	16 42.9	+68 15	7.8	G4	+ 7 + 3 + 11	+ 7	+ 22	0.47	328	+0.05	119	+52	39	6.2
Lal. 30694	16 48.4	+ 0 10	6.8	G5	+ 38 + 45 + 41	+ 41	+ 57	1.66	206	+0.07	215	−51	117	5.9
Lal. 31132	17 0.1	+47 12	6.7	G7	− 44 − 47 − 46	− 46	− 27	0.83	9	+0.14	61	− 8	36	7.4

TABLE II—*Continued*

Star	R. A. 1910	Dec. 1910	Mag.	Spec.	v from Separate Plates	Mean v	v'	μ	ψ	π	A	Δ	V	Abs. Mag.
					km	km	km						km	
Brad. 2179	17h10m7	−26 25	6.7	K3	− 7 − 12 − 13	− 11	0	1.″26	204°	+0.″30	158°	−22°	5	9.1
72 *w Herculis*	17 17.3	+32 35	5.4	Go	− 80 − 77 − 77 − 80	− 78.4	− 58.7	1.05	174	+0.12	63	−68	72	5.8
Fed. 2895	17 25.3	+67 24	6.3	K1	− 41 − 38 − 35	− 38.3	− 22.5	0.53	273	+0.05	171	−27	54	4.7
Brad. 2388	18 53.7	+32 48	5.2	G1	− 41 − 44 − 46	− 43.4	− 23.8	0.23	135	+0.02	11	−53	57	.....
Groom. 2789 *N.*	19 9.8	+49 40	6.8	G9	− 44 − 40 − 34 − 38	− 39	− 21	0.62	348	+0.02	124	+31	144	.....
Groom. 2789 *S.*	19 9.8	+49 40	6.6	G9	− 43 − 40 − 39 − 41	− 41	− 23	0.62	348	+0.02	124	+30	144	.....
31 *b Aquilae*	19 20.7	+11 45	5.2	G7	− 96 − 97 −100	− 97.6	− 79.6	0.96	49	+0.06	80	+21	110	4.3
3 *Cygni*	19 21.7	+24 45	6.2	F4	− 4 − 7 − 5	− 5.0	+ 13.9	0.66	196	+0.07	271	−46	47	5.3
Groom. 2875	19 29.7	+55 24	6.7	Ko	+ 10 + 14 + 11	+ 12	+ 30	0.66	232	−0.06	.....	.....	.....	.....
Lal. 37120–1	19 30.1	+33 0	6.6	G2	−162 −162 −164 −161	−162	−143	0.48	295	+0.06	310	+26	150	5.4
16 *Cygni Pr.*	19 39.4	+50 19	6.3	G3	− 28 − 27 − 24	− 26.1	− 8.4	0.22	228	+0.16	216	−46	17	7.3
Lal. 38287	19 58.6	+15 23	7.2	G6	+ 9 + 11 + 15	+ 12	+ 29	0.60	198	+0.09	272	−22	42	6.9
Lal. 38380	19 59.9	+29 40	5.7	Ko	− 47 − 44 − 48	− 46.4	− 28.4	0.86	128	+0.04	25	−43	98	3.5
15 *Sagittae*	20 0.0	+16 50	5.9	Go	+ 2 + 5 + 3	+ 3.1	+ 20.3	0.56	227	+0.10	250	−10	37	6.0
Lal. 38383	20 0.1	+23 7	7.2	G8	+ 3 − 7 − 3	− 2	+ 16	1.39	227	+0.03	232	−34	224	.....
Groom. 3042	20 3.8	+52 53	5.7	F5	− 43 − 45 − 37 − 39	− 41	− 24	0.33	42	+0.06	102	−23	29	4.5
Pi. 20h23	20 7.0	+15 56	7.3	G7	− 49 − 52 − 51	− 50	− 33	0.55	311	+0.08	161	+18	54	6.7
Lac. 8381	20 9.7	−27 18	5.7	K6	− 58 − 57 − 60	− 58	− 50	1.27	101	+0.01	(41	− 6	592)	.....
Groom. 3150	20 16.5	+66 33	6.1	F8	− 11 − 4 − 4	− 6.1	+ 8.8	0.57	57	+0.12	32	+43	14	6.4
Groom. 3215	20 29.8	+41 34	7.0	G8	− 13 − 13 − 8	− 11	+ 6	0.48	340	+0.01	.....	.....	.....	.....
Groom. 3263	20 38.4	+60 11	6.0	Fo	− 10 − 11 − 14	− 11.6	+ 3.7	0.19	3	+0.02	.....	.....	.....	.....
Groom. 3357	20 56.5	+39 53	6.6	F6	− 35 − 36 − 41	− 37	− 21	0.33	47	+0.08	129	− 8	25	6.1
W.B. 21h97	21 7.9	+17 23	7.3	F5	− 48 − 46 − 43	− 46	− 32	0.91	188	−0.12	.....	.....	.....	.....
Lac. 8777	21 14.6	−26 44	6.5	G5	− 46 − 40 − 44	− 43	− 37	0.68	238	−0.10	.....	.....	.....	.....
Brad. 2792	21 22.1	+46 20	5.5	A2	− 5 + 5 − 2 + 50	+ 1	+ 16	0.20	72	+0.09	300	+52	16	5.2
24 *Aquarii*	21 34.9	− 0 27	6.8	F8	− 20 − 16 − 16 − 17	− 17	− 7	0.24	82	−0.03	.....	.....	.....	.....
Lal. 42883–5	21 54.8	+29 24	6.9	F7	+ 6 + 11 + 5 + 5	+ 7	+ 20	0.56	223	+0.03	270	−25	98	.....
Lal. 43492	22 12.7	+12 26	6.9	Go	− 28 − 35 − 29	− 31	− 21	0.84	84	+0.02	171	+ 7	186	.....
Pi. 22h214	22 41.5	+29 58	6.5	Ko	− 5 + 5 − 1 − 2	− 1	+ 9	0.47	214	−0.02	.....	.....	.....	.....
Fed. 4371	23 1.5	+67 56	7.5	G2	− 23 − 22 − 20	− 21	− 10	0.60	75	+0.06	102	− 8	33	6.2
Brad. 3077	23 8.9	+56 40	5.6	K4	− 20 − 18 − 25 − 19	− 20.4	− 9.9	2.11	82	+0.16	275	− 2	47	6.6
Lal. 45755	23 17.3	+43 34	7.6	G7	+ 2 + 3 0	+ 2	+ 11	0.67	71	+0.04	92	+27	67	5.5
Pi. 23h164	23 38.9	+57 33	7.0	F8	− 65 − 69 − 71	− 68	− 59	0.62	38	−0.08	.....	.....	.....	.....

values obtained have been checked by means of the following equations which are due to Professor Kapteyn:

$$x=v\cos\delta\cos\alpha-4.74\frac{\mu\sin\psi}{\pi}\sin\alpha-4.74\frac{\mu\cos\psi}{\pi}\sin\delta\cos\alpha$$

$$y=-17.3+v\cos\delta\sin\alpha+4.74\frac{\mu\sin\psi}{\pi}\cos\alpha-4.74\frac{\mu\cos\psi}{\pi}\sin\delta\sin\alpha$$

$$z=+10.0+v\sin\delta+4.74\frac{\mu\cos\psi}{\pi}\cos\delta$$

$$V^2=x^2+y^2+z^2.\quad \tan A=\frac{y}{x}.\quad \sin\Delta=\frac{z}{V}.$$

(sin A has the same sign as y)

The uncertainty in the value of the parallax of many of these stars of course affects very greatly the determination of their motions in space, especially when the parallax is small. Accordingly, when the direction and amount of motion of a star with a parallax of but a very few hundredths of a second are given, they are to be considered rather as representing what the values would be in case the observed parallax is assumed to be correct, and not necessarily the actual motion of the star.

The number of measures of each plate by different observers varies between one and five, the great majority being either two or three. All of the plates without exception have been measured by Miss Lasby, and a large proportion by Miss Ensign and by Adams. Miss Burwell has measured many of the more recent spectrograms, and some of the earlier results are due to Miss Waterman.

The computations have been made almost entirely by Miss Lasby, to whom we wish to express our hearty appreciation.

In addition to the values given in Table II the following determinations of the radial velocities of stars observed elsewhere are added for comparison:

TABLE III

Star	Mag.	Spec.	v		
			Separate Plates	Mean	Campbell
Groom. 1830	6.5	G5	−99 −96 −96	−97.1	−97
61^1 *Cygni*	5.6	K8	−65 −66 −65 −65	−65.2	−63
61^2 *Cygni*	6.3	K8	−69 −68 −62 −61	−65	

Some of the principal conclusions to be drawn from an inspection of the results shown in Table II are as follows:

1. A few stars show enormous radial velocities. Chief among these are *Lal.* 1966 and *Lal.* 15290, with velocities of −325 and −242 km. The first of these shows the highest radial velocity so far observed among any of the stars. In addition to these there are four stars showing velocities of over 100 km, and several with velocities between 75 and 100 km.

2. A peculiar fact is the great preponderance of large negative over large positive velocities. If we use for this comparison the velocities corrected for the sun's motion, given as v' in the table, and set 50 km as a limit, we obtain the following:

Positive (5)			Negative (15)					
Groom. 864	G0	+100	*Lal.* 1045	K1	−58	*W.B.* $15^{h}720$	G9	−54
Groom. 1281	F9	84	*Lal.* 1966	F3	319	*Lal.* 28607	A2p	158
20 *Leo Min.*	G1	54	*Lal.* 4855	G0	103	72 *w Herculis*	G0	59
33 *Virginis*	K1	56	*Lal.* 5761	A3p	151	31 *b Aquilae*	G7	80
Lal. 30694	G5	+57	*Lal.* 15290	F7	250	*Lal.* 37120–1	G2	143
			Lal. 21185	Ma	85	*Lac.* 8381	K6	50
			Lal. 27744	G9	58	*Pi* $23^{h}164$	F8	−59
			OΣ 298	K0	−55			

Accordingly no less than 75 per cent of the large velocities observed are negative.

3. An examination of the spectral types of the stars with great velocities shows that nearly all classes are represented among them. An interesting fact, however, is that the two stars with the largest velocities of all are of types F3 and F7, and the two stars succeeding these are of the A type. The spectra of these last stars, *Lal.* 5761 and *Lal.* 28607, are both peculiar in that the magnesium line λ 4481 is either absent or extremely faint; while in other respects the spectra are of the normal A2 or A3 type. The remarkably high and nearly equal velocities for two stars of nearly identical but peculiar spectra form a singular coincidence. The stars are widely apart in the sky, and the apices of their motions are quite different. The existence of high radial velocities among stars having what is generally considered an early type of spectrum is shown by these results, although there can be no doubt that such cases are rare. Professor Campbell found no certain case of a star

with a constant velocity exceeding 40 km in an investigation of 337 stars having spectra between types B and F4,[1] and in the course of our observations of about 350 stars of types B and A we have found but one star with a very large constant velocity. This star is 7 *Sextantis, Boss* 2647, and four determinations of its velocity give values of +97, +100, +93, and +94 km, with a mean value of +96 km. Its spectrum is A3 and normal in all respects. The proper motion is exceptionally large as well.

4. Two interesting stars in the list are *Groom.* 34 and *Lal.* 21185, both of which have very large proper motions and parallaxes. The motion of *Groom.* 34 is almost entirely across the line of sight, but *Lal.* 21185 shows the high radial velocity of −87 km. These stars are among the faintest stars known, having absolute magnitudes of 10.4 and 10.6 respectively, when the unit of distance is taken for a parallax of 0".01, or the sun with a magnitude of 5.5. Their spectra are both of type Ma.

5. The physical connection of two pairs of double stars in the list is almost certainly established by the radial velocities. These are *Lal.* 18115, where the components have a separation of 20", and *Groom.* 2789, with a separation of 10". The spectra of the two components are identical in each case. These stars belong to what is frequently known as the 61 *Cygni* type of double stars. A peculiar case is that of OΣ 298 and *W.B.* 15^h720, which are distant over 2' from each other, but show common proper motions and equal radial velocities of −71 km. By a singular coincidence the direction of motion of these stars in space is almost exactly opposite to that of the sun. Of the other stars in the list only one, 32 *Lyncis*, has a direction of motion which is approximately parallel to that of the sun.

MOUNT WILSON SOLAR OBSERVATORY
December 8, 1913

[1] *Stellar Motions*, p. 198.

SOME PYRHELIOMETRIC OBSERVATIONS ON MOUNT WHITNEY

By A. K. Ångström and E. H. Kennard

In the summer of 1913 an expedition supported by a grant from the Smithsonian Institution proceeded to California in order to study the nocturnal radiation under different atmospheric conditions. In connection with these investigations we had an opportunity to measure the intensity of the solar radiation during seven clear days on the summit of Mount Whitney (4410 m). These measurements were made for different air masses and include observations of the total radiation and of the radiation in a special part of the spectrum, selected by means of an absorbing screen, as has been proposed by K. Ångström.[1] Our paper will present the results of the observations and a computation from them of the solar constant.

INSTRUMENTS

The observations were made with Ångström's pyrheliometer No. 158. With this instrument the energy of the radiation falling upon the exposed strip is given in calories per square centimeter per minute by the relation $I=kC^2$, where C is the compensating current sent through the shadowed strip, and k is a constant which was determined for this instrument at the solar observatory of the Physical Institute in Upsala and found to be 13.58.[2] The compensating current was furnished by 4 dry cells, which proved entirely suited to the purpose. It was measured by a Siemens and Halske milliammeter. For further details of the instrument and the method of using it, we refer to the original paper.[3]

The absorbing screen, used in order to study a limited part of the spectrum, was composed of a water cell, in which the water layer had a thickness of 1 cm, and a colored glass plate, Schott

[1] *Nova Acta Reg. Soc., Sc. Upsal.*, Ser. IV, **1**, No. 7.

[2] A comparison made at the Smithsonian Institution in Washington showed that the readings of this instrument are 4.57 per cent lower than the Smithsonian scale.

[3] *Astrophysical Journal*, **9**, 332, 1899.

and Genossen, 436[III], the thickness of which was 2.53 mm. The transmission of the combination for different wave-lengths as previously determined at Upsala by Mr. A. K. Ångström is given in Fig. 1. The maximum of transmission occurs at wave-length 0.526 μ, and 85 per cent of the transmitted light is included between 0.484 μ and 0.576 μ.

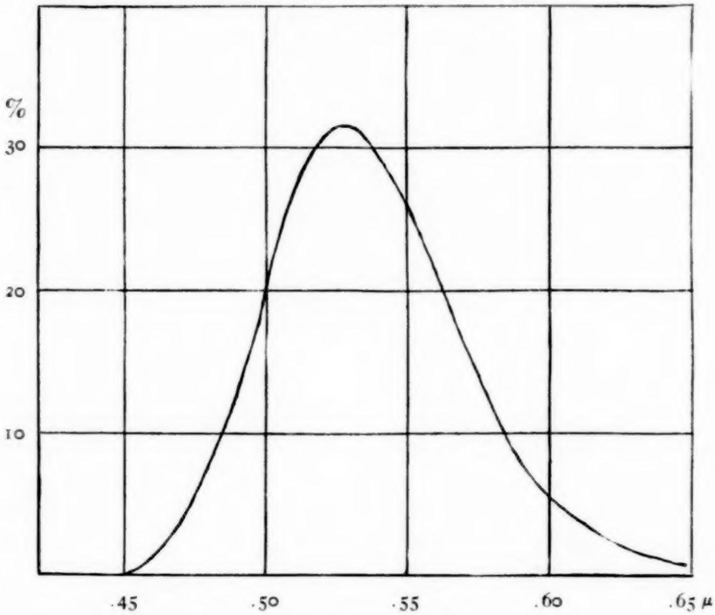

FIG. 1.—Transmission curve of absorbing screen

The local time of each observation, from which the sun's zenith angle and finally the corresponding air mass was computed, was determined from the readings of three watches. Before and after the expedition to Mount Whitney, the watches were compared with the daily telegraphic time signal at Claremont, California. The time is probably accurate within half a minute.

RESULTS

The results are given in Tables I and II. Table I refers to the measurements of the total radiation and contains: (1) the date, (2) the local apparent time, (t), (3) the computed air mass, (m),

(4) readings of the milliammeter, (s), (5) the total radiation computed from the readings. Table II contains the same quantities relating to measurements taken with the absorbing screen.

Bemporad's[1] expression for the air mass in terms of the apparent zenith angle was employed. His values for 60°, 70°, 80°, and 85° were available in a short table given by F. Lindholm.[2] The differences between these values and the secant of the angle give

TABLE I

MEASUREMENTS OF TOTAL RADIATION

	t	m	s Milliamp. ×½	Q_m $\frac{\text{cal.}}{\text{cm}^2\,\text{min.}}$
August 2	6ʰ34ᵐ2	3.337	100.1	1.224
	6 49.2	2.872	102.5	1.287
	7 30.7	2.088	106.3	1.381
	8 13.2	1.657	108.8	1.446
	9 20.7	1.299	111.3	1.514
August 4	6 28.3	3.630	99.4	1.202
	6 58.8	2.672	104.0	1.322
	7 6.8	2.501	104.1	1.325
	8 4.3	1.741	108.6	1.441
	9 6.8	1.359	110.5	1.493
	11 0.3	1.089	111.7	1.520
	11 8.8	1.081	112.0	1.533
August 5, A.M.	6 29.5	3.608	97.8	1.169
	7 2.0	2.616	103.0	1.296
	7 48.0	1.906	107.0	1.399
	8 59.0	1.397	110.6	1.495
	10 0.5	1.190	111.1	1.508
August 5, P.M.	2 0.3	1.193	111.2	1.511
	3 3.3	1.410	109.5	1.465
	4 4.3	1.830	106.3	1.381
	4 33.8	2.185	104.2	1.326
	5 4.8	2.783	100.1	1.224
	5 24.3	3.377	96.6	1.141
August 10	6 33.0	3.630	95.6	1.117
	7 3.0	2.681	100.5	1.235
	7 56.5	1.857	105.5	1.360
August 11	6 27.1	3.952	96.9	1.147
	6 54.6	2.914	101.9	1.269
	7 40.1	2.053	106.0	1.373
	8 41.6	1.514	109.3	1.460
	10 13.1	1.177	111.7	1.525
August 12	6 26.6	4.018	98.1	1.176
	6 59.1	2.817	103.6	1.312
	7 55.1	1.889	108.5	1.439
	8 57.1	1.435	111.1	1.509
	10 43.6	1.127	113.0	1.561

[1] *Mitteilungen der Grossherzoglichen Sternwarte zu Heidelberg*, No. 4, 1904.

[2] *Nova Acta Reg. Soc., Sc. Upsal.*, Ser. IV, **3**, No. 6.

TABLE II

MEASUREMENTS WITH ABSORBING SCREEN

	t	m	s Milliamp. ×2	Q_m $\frac{\text{cal.}}{\text{cm}^2 \text{ min.}}$
August 2	6^h18^m2	4.044	104.5	0.0371
	6 54.7	2.733	114.1	0.0442
	7 25.7	2.158	122.0	0.0505
	8 22.7	1.589	125.4	0.0534
	8 31.7	1.530	126.8	0.0546
	9 15.2	1.319	128.8	0.0562
August 4	6 16.8	4.204	103.1	0.0361
	6 36.3	3.316	112.1	0.0426
	7 11.8	2.406	118.9	0.0480
	8 9.3	1.699	125.3	0.0533
	9 19.3	1.311	128.0	0.0556
	11 13.3	1.077	129.9	0.0573
August 5, A.M.	6 17.0	4.237	101.8	0.0352
	6 36.0	3.352	108.8	0.0402
	8 3.5	1.755	123.1	0.0515
	9 5.5	1.368	127.9	0.0554
	10 7.0	1.175	129.4	0.0568
August 5, P.M.	2 6.8	1.209	129.3	0.0567
	3 12.8	1.457	126.7	0.0545
	4 11.8	1.907	122.4	0.0509
	4 40.3	2.287	118.3	0.0475
	5 10.3	2.928	114.1	0.0441
	5 30.3	3.615	106.6	0.0386
August 9	6 14.4	4.607	96.0	0.0313
	6 33.9	3.559	103.4	0.0363
	11 38.9	1.126	128.8	0.0563
August 10	6 21.5	4.211	100.6	0.0344
	6 38.0	3.428	106.7	0.0387
	7 8.0	2.570	113.8	0.0439
	8 2.0	1.804	122.4	0.0508
	8 6.0	1.767	122.0	0.0505
August 11	6 14.6	4.716	102.5	0.0356
	6 33.6	3.641	107.9	0.0395
	7 0.1	2.770	114.6	0.0445
	7 45.1	1.992	122.1	0.0507
	8 51.1	1.462	127.1	0.0549
	10 18.6	1.166	129.9	0.0573
August 12	6 13.1	4.895	99.1	0.0333
	6 34.1	3.656	108.0	0.0397
	7 5.1	2.671	116.4	0.0459
	8 3.6	1.804	123.5	0.0517
	9 2.6	1.409	128.1	0.0557
	10 52.6	1.115	131.5	0.0587

the (negative) corrections to be applied to the secants of these angles. Through these values of the correction an algebraic curve of 4 terms was passed and the correction was then calculated for other angles. In obtaining the apparent zenith angle, allowance was made for refraction.

GENERAL DISCUSSION OF THE EMPIRICAL METHODS FOR COMPUTING THE SOLAR CONSTANT

Empirical methods for determining the solar constant from pyrheliometric measurements alone have been proposed by K. Ångström[1] and by Fowle.[2] Both these methods are based upon results obtained from spectrobolometric observations. Ångström's method assumes that from Abbot and Fowle's observations we know both the distribution of energy in the sun's spectrum and the general transmission of the atmosphere for all wave-lengths in terms of its value for any given wave-length. It assumes further that the absorption caused by the water-vapor is a known function of the water-vapor pressure at the earth's surface; for this Ångström proposed an empirical formula based upon his spectrobolometric curves. The influence of diffusion and absorption can then be calculated if the transmission for some chosen wave-length is known from pyrheliometric observations on a limited part of the spectrum.

Fowle's method is much briefer. He plots the logarithms of the observations against the air masses and extrapolates to air-mass zero by means of the straight line that best fits the points. To the "apparent solar constant" thus obtained he applies an empirical correction depending upon the locality, and derived from local spectrobolometric observations.

Since these methods are founded upon the spectrobolometric method, one may ask, what is the justification for using them instead of the latter? Can they be expected to give something more than the method upon which they are founded? To the first question one may reply that the justification lies in their simplicity, which makes it possible to apply them under a wide range of conditions where the more cumbersome bolometric method could never be used. A spectrobolometric investigation, like that of Abbot on Mount Whitney in 1910, will probably always be a rare event. But especially in regard to the question of solar variability it is desirable that the number of simultaneous observations be large and extended to as high altitudes as possible.

[1] *Nova Acta Reg. Soc., Sc. Upsal.*, Ser. IV, **1**, No. 7.

[2] *Annals of the Astrophysical Observatory, Smithsonian Inst.*, **2**, 114.

The second question, whether the abridged methods can ever deserve the same confidence, or even in rare cases give greater accuracy then the spectrobolometric observations, is one that must be answered rather through experimental results than through general considerations. Here, however, two points may be noted.

The first is, that the spectrobolometric method, which under ideal conditions is naturally superior to any abridged method, is in all practical cases a method involving a large number of precautions, some of which are very difficult to take. The abridged methods, founded as they are upon mean values, may possibly under special conditions avoid accidental errors to which single spectrobolometric series are subjected.

Secondly, it may be noted, that even in the analytical method of bolometry, there arises some uncertainty in regard to the ordinates of the bolometric curve, corrected for absorption, at the points where absorption bands are situated. This causes an uncertainty in the water-vapor correction in this method as well as in the abridged methods founded upon it.

The methods just discussed lead to a numerical value for the solar constant. But the measurements in a selected part of the spectrum lead also to a direct test of solar variability, which seems likely to be especially valuable because these observations are not affected by aqueous absorption.

MEASUREMENTS WITH ABSORBING SCREEN

We may put:

$$I = I_0 e^{-\gamma m}$$

where I_0 is the energy transmitted through the absorbing screen at the limit of the atmosphere, I is its value after passing through the air mass m, and γ is a constant dependent upon the scattering power of the atmosphere. If now we plot log I against m, the points should lie on a straight line, whose ordinate for $m = 0$ is log I_0.

The values of I_0 thus obtained from our observations are given under the heading I_0 in Table III. The straight lines were run by the method of least squares, not so much because the presuppositions of this method seemed here to be satisfied, as because thereby

all personal bias was eliminated. The "probable error" e of each value of I_0 is appended as a rough indication of its reliability, and the weighted mean I_0 is given at the bottom of the table. A comparison between the different values of I_0 shows that they all differ by less than 2 per cent; half of them by less than $\frac{1}{2}$ per cent, from the mean value. The deviation falls as a rule within the limits of the probable error.

This result thus fails to support the variability of the sun inferred by Abbot from simultaneous observations at Bassour and Mount Wilson. We cannot, however, with entire safety draw any conclusions about the total radiation from measurements in a limited part of the spectrum. All that can be said with certainty is that *a change of the energy in the green part of the solar spectrum exceeding 2 per cent during the period of our observations is improbable.*

If we, from this, are inclined to infer that the total solar radiation during the same period was constant, this inclination rests upon a statement by Abbot[1] himself to this effect: "So far as the observations[2] may be trusted, then, they show that a decrease of the sun's emission of radiation reduces the intensity of all wave-lengths; but the fractional decrease is much more rapid for short wave-lengths than for long."

Yet unpublished measurements by Mr. A. K. Ångström, in Algeria at 1160 m altitude, give a mean value for I_0 equal to 0.0708 which is in close agreement with the value 0.0702 given above. On the former occasion Mr. Abbot's spectrobolometric observations gave a mean value for the solar constant of 1.945. If we assume the energy transmitted by our green glass on Mount Whitney to bear the same ratio to the total energy, the Mount Whitney observations give a value for the solar constant reduced to mean solar distance equal to 1.929, which differs by less than 1 per cent from the former value.

MEASUREMENTS OF THE TOTAL RADIATION

The general basis of the Ångström-Kimball method of calculation has already been described. It is here convenient to make

[1] *Annals of the Astrophysical Observatory, Smithsonian Institution*, **3**, 133, 1913.

[2] *Observations at Bassour and Mount Wilson*, 1911–1912.

use of the spectrum of constant energy introduced by Langley, where the abscissa represents the energy included between an extreme (ultra-violet) wave-length and the wave-length corresponding to the abscissa; the energy-density plotted as ordinate would then be constant. A table giving wave-lengths and corresponding abscissae is given by Kimball.[1]

Referred to such a spectrum, the atmospheric transmission y_x for any wave-length is well represented by the empirical formula

$$y_x = p^{m\delta} x^{nm\phi(\delta)} \qquad (1)$$

where x is the abscissa, m the air mass, and δ a quantity dependent upon the scattering power of the atmosphere. Ångström made the natural assumption $\phi(\delta)=\delta$. Kimball[2] finds that $\phi(\delta)=\sqrt{\delta}$ better fits the observations at Washington and Mount Wilson. In the latter case we have,

$$p = 0.93 , \qquad n = 0.18$$

Making these substitutions in (1) and integrating,

$$Q_m = Q_0 \int_0^1 0.93^{m\delta} x^{0.18m\sqrt{\delta}} dx$$

or

$$Q_m = Q_0 \frac{0.93^{m\delta}}{1+0.18m\sqrt{\delta}} .$$

Kimball then adds an empirical correction for the absorption due to water-vapor, based upon bolometric measurements at Washington and at Mount Wilson, and finally obtains

$$Q_0 = \frac{Q_m}{\dfrac{0.93^{m\delta}}{1+0.18m\sqrt{\delta}} - [0.061 - 0.008\delta + 0.012 E_0 m]} \qquad (2)$$

where E_0 represents the depth in millimeters to which the earth's surface would be covered by water if all the aqueous vapor were precipitated. We have adopted this expression, *but instead of attempting to determine E_0 from humidity measurements at the*

[1] *Bulletin of the Mount Weather Observatory*, **1**, Parts 2 and 4.

[2] *Ibid.*

earth's surface we have eliminated it between two equations such as (2) involving different air masses.

Kimball eliminates δ between two such equations. We have, however, followed the original method of K. Ångström and have determined δ for each day from our measurements with the green glass. The energy maximum of the light transmitted by it lies at $0.526\,\mu$ (see Fig. 1) to which corresponds the abscissa 0.27 in the constant energy spectrum. Hence for the transmitted green light

$$I_m = I_0 0.93^{m\delta} 0.27^{0.18m\sqrt{\delta}}$$

from which δ can be computed. The values of δ thus obtained are given in Table III.

In order to compute Q_0, a smooth curve was drawn through the observations and values of Q_m for $m=1$, 2, and 3 were read off from the curve. These values and the value of δ for the day were inserted in (2) and E_0 then eliminated between the first and second and the first and third of the equations thus obtained. The results are given in Table III under the headings Q_{12}, Q_{13}; the mean of these for each day is given under $Q_{KÅ}$ and represents the solar constant as obtained for that day by the Ångström-Kimball method.

The mean value of all the measurements, reduced to mean solar distance, is $1.931 \frac{\text{cal.}}{\text{cm}^2\,\text{min.}}$ (Ångström scale) or 2.019 (Smithsonian scale). The maximum deviation from the mean is 3 per cent.

Finally, Fowle's abridged method was applied to the same observations. Sufficient observations are not available for the elaboration of a special correction suited to Mount Whitney. But from the values of δ, it appears that the transmission over Mount Whitney was about the same as over Mount Wilson, where the average value of δ is 0.25; and the water-vapor pressure, the most uncertain factor, was low (2–4 mm). Hence it may not be devoid of interest to apply here Fowle's rule as elaborated for Mount Wilson, which is: To the "apparent solar constant" obtained by straight-line extrapolation add 2.7 per cent and as many per cent as there are millimeters in the water-vapor pressure. The results thus obtained are given in Table III under the heading Q_F; the mean water-vapor pressure is given under p.

TABLE III

FINAL RESULTS

	P mm	δ	I_0 $\frac{\text{cal.}}{\text{cm}^2\text{ min.}}$	e per cent	Q_{11} $\frac{\text{cal.}}{\text{cm}^2\text{ min.}}$	Q_{13} $\frac{\text{cal.}}{\text{cm}^2\text{ min.}}$	$Q_{K\mathring{A}}$ $\frac{\text{cal.}}{\text{cm}^2\text{ min.}}$	Q_F $\frac{\text{cal.}}{\text{cm}^2\text{ min.}}$
August 2......	(3.0?)	0.30	0.0689	0.9	1.904	1.886	1.895	(1.820)
August 4......	3.0	0.28	0.0678	0.9	1.847	1.829	1.838	1.793
August 5, A.M..	2.5	0.32	0.0683	0.3	1.871	1.874	1.873	1.832
August 5, P.M..	2.9	0.32	0.0684	0.8	1.887	1.900	1.894	1.878
August 9......	......	(0.39)	(0.0688)	......	......	......	......	......
August 10......	3.4	0.33	0.670	0.7	......	1.826	(1.826)	(1.770)
August 11......	2.2	0.30	0.685	0.5	1.877	1.870	1.874	1.793
August 12......	2.0	0.29	0.685	0.5	1.896	1.888	1.892	1.802

$$\text{Weighted mean } I_0 = 0.0683 \frac{\text{cal.}}{\text{cm}^2\text{ min.}} ,$$

$$\text{reduced to mean solar distance } I_0 = 0.0702 \frac{\text{cal.}}{\text{cm}^2\text{ min.}}$$

(Ångström scale)

$$\text{Mean reduced to mean solar distance: } Q_{K\mathring{A}} = 1.931(\text{Å.}) ,$$

$$= 2.019(\text{Sm.}) \frac{\text{cal.}}{\text{cm}^2\text{ min.}}$$

$$Q_F = 1.872(\text{Å.}) ,$$

$$= 1.960(\text{Sm.}) \frac{\text{cal.}}{\text{cm}^2\text{ min.}}$$

SUMMARY

Our pyrheliometric observations on the top of Mount Whitney, extending from August 2 to August 12, 1913, have led to the following results:

1. A variation in the solar constant amounting to more than 2 per cent during this time is improbable.

2. The solar constant computed from the measurements in a selected part of the spectrum, reduced to mean solar distance, came out 1.929 $\frac{\text{cal.}}{\text{cm}^2\text{ min.}}$ (Smithsonian scale), with a possible error of 1.5 per cent. This value is obtained on the assumption that the energy included between 0.484 μ and 0.576 μ is a constant known fraction of the total energy in the solar spectrum.

3. The solar constant computed by the Ångström-Kimball method was found to be 2.019 $\frac{\text{cal.}}{\text{cm}^2\text{ min.}}$ (Smithsonian).

4. The solar constant computed according to Fowle's method comes out $1.960 \frac{\text{cal.}}{\text{cm}^2 \text{ min.}}$ (Smithsonian).

The value of the solar constant given in (2) is in close agreement with Abbot's mean value of 1.932 obtained from several series of observations made during the years 1902–1912 at much lower altitudes (e.g., at 1160 m in Algeria). The value given in (3) is also in close agreement with the solar constant computed by Kimball according to the same method from measurements at Washington. Consequently our observations give no support to a value of the solar constant greatly exceeding $2 \frac{\text{cal.}}{\text{cm}^2 \text{ min.}}$.

Because of their bearing upon the question of solar variability, it seems desirable that the observations in selected parts of the spectrum by means of absorbing screens should be extended to different localities, and that if possible simultaneous measurements at elevated stations should be undertaken.

CORNELL UNIVERSITY
December 1913

THE COLOR OF THE FAINT STARS[1]

By FREDERICK H. SEARES

During the last two or three years evidence that the faint stars are appreciably redder than the brighter objects has gradually been accumulating. Kapteyn has found, for example, that certain clusters are distinctly redder than bright stars whose spectra are the same as the average spectrum of the clusters with which they were compared. A similar result, although complicated with photographic phenomena, is indicated by Mount Wilson photographs of faint stars made with red and blue filters. Perhaps the most recent contribution is that of Hertzsprung,[2] who has photographed numerous regions with a large grating attached to the tube of the 60-inch reflector. The results for *N.G.C.* 1647 are striking. From the ninth magnitude on there is a gradual increase in the effective wave-length. At magnitude 14.5 its minimum value is λ 4320. If this result is interpreted in terms of spectral type, it means that for the region considered there are no stars of this magnitude whose spectra are earlier than F0. Other considerations which need not be discussed here all point in the same direction.

It is important that the question should be investigated further, for other regions and for still fainter stars, and preferably by independent methods. A simple method of attack is the direct formation of color indices by comparing visual and photographic magnitudes. If the faint stars are redder than the bright stars, this fact must immediately be revealed by a difference in the average index for the two groups. Should blue or white stars be rare or altogether lacking among the fainter objects, we shall find no negative and possibly no small positive color indices.

Although simple in principle, the method is exacting in its demands, for both visual and photographic magnitudes must be determined independently in accordance with an absolute scale.

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 81. Read at the sixteenth meeting of the Astronomical and Astrophysical Society of America, Atlanta, January 1914.

[2] "Annual Report of the Director of the Mount Wilson Solar Observatory," *Year Book of the Carnegie Institution of Washington*, No. 12.

Moreover, careful attention must be given to the zero points, as an error in either of those enters directly into the color index. So far as a change in color with increasing magnitude is concerned, such an error is of no consequence. But to determine the actual degree of color, the zero points must be established in a definite relation to each other—preferably that already fixed by international convention, which requires that the photographic and visual magnitudes of stars of spectrum A0 between 5.5 and 6.5 on the Harvard scale shall be equal. Since we desire color indices for very faint stars—let us assign temporarily magnitude 17.5 as a limit—we must establish absolute photographic and visual scales over an interval of at least twelve magnitudes.

A natural point of beginning is the North Polar Sequence, for which two determinations of the photographic scale are already available—one made under the direction of Professor Pickering at the Harvard Observatory, the other from observations at Mount Wilson. With the exception of a divergence of about 0.4 mag. between the sixth and the tenth magnitudes, the agreement is very satisfactory. In addition, Pickering has determined the visual magnitudes of the stars brighter than about thirteen. The essential thing remaining, therefore, is the extension of the visual scale to the fainter stars.

This has been done photographically with the 60-inch reflector, using isochromatic plates and a yellow filter. The scale has been established by diaphragms, the methods of observation and reduction being the same as those previously employed for the determination of the photographic scale.[1] The zero point was determined from nine stars between 9.84 and 13.94 for which visual magnitudes are given in *H.C.*, No. 170. As a control, the bright stars of the sequence between the fifth and seventh magnitudes were directly connected with fainter stars between magnitudes twelve and thirteen. Here the method was the same as that used in deriving the photographic magnitudes for the brighter stars described in *Contribution* No. 70. The results of the control observations were most satisfactory, and confirm the relation of

[1] *Contributions from the Mount Wilson Solar Observatory*, Nos. 70, 80; *Astrophysical Journal*, **38**, 241, 1913; **39**, 307, 1914.

the Harvard visual magnitudes between twelve and thirteen to those near the sixth magnitude. There can scarcely be any question, therefore, as to the substantial accuracy of the visual scale as far as the thirteenth magnitude. Beyond this point, as already stated, the same methods were used as had been successfully employed for the photographic scale of the fainter stars. The separate determinations, of which there are five, are all closely accordant, and aside from the relatively small amount of observational data, there seems to be no more reason to fear systematic errors here than there.

Briefly, the question of scales stands thus, reference throughout being to the international zero point. For the brighter stars there is a linear divergence between the Harvard and Mount Wilson photographic scales, which at the tenth magnitude amounts to 0.37 mag. From 10 to 15.5 the scales are parallel, the Harvard magnitudes being brighter than the Mount Wilson values. Beyond 15.5 there is again a small divergence, unimportant for the moment, as it does not affect the general character of the result. The Harvard visual magnitudes from 12 to 13 are confirmed in their relation to the international zero point and the Mount Wilson extension of the visual scale to the seventeenth magnitude is presumably reliable, as it has been established by tested methods. Any change in the average calculated color index between the tenth and seventeenth magnitudes should therefore be reliable to the same degree, although the absolute amount of the index may be uncertain by reason of the constant difference which arises from the divergence between the sixth and tenth magnitudes previously referred to. The question of this difference will be discussed presently.

In the meantime attention is directed to the upper portion of Fig. 1 which illustrates the distribution of the color indices for the region of the Pole. In order that the results might be representative, as many stars as possible, 107 in all, were included. The distances of the points from the axis are proportional to the indices of the individual stars whose photographic magnitudes are indicated in the margin. In calculating the indices Mount Wilson photographic magnitudes were used throughout. For stars brighter

than ten the visual magnitudes were taken from *H.C.*, No. 170. For all the fainter stars Mount Wilson photovisual magnitudes were used.

A possible variation in the average color index is obscured by the sporadic appearance of stars of high intrinsic color; but the gradual and regular increase in the minimum value indicated by the curve bounding the lower edge of the field of points is clearly indicated. From the sixth to the seventeenth magnitudes the change is from -0.1 to $+0.6$ mag. In reality the variation probably is greater, for the brighter objects form a selected group. They include only polar sequence stars, and, with the exception of a few red stars, are almost wholly of the A type. No early B-type stars appear; but in considering the change in the minimum index we must take them into account, for their non-appearance is doubtless due to a restriction in the choice of objects to be used as standards. On the other hand, the faint stars are so numerous that they must be fairly representative, and it is improbable that any increase in the field would modify their minimum index. Since the color index of bright B0 stars is about -0.4 mag., the real change in the minimum index between the sixth and seventeenth magnitudes is probably about one magnitude. Beyond the fifteenth magnitude there appear no stars with indices less than $+0.5$ mag. Inferentially their spectra would be at least F2. Had the Harvard photographic magnitudes been used, the bounding curve would have coincided with the axis for the brighter objects, since the Harvard visual and photographic scales coincide for A-type stars; below the tenth magnitude the two photographic scales are parallel, and the characteristic variation would have appeared, although all the color indices would have been smaller by 0.37 mag.

This very considerable modification of the result raises again the question of the divergence of the Harvard and Mount Wilson photographic scales for the brighter stars. As a possible source of explanation my attention has been called to the following sentence in *H.C.*, No. 170, relating to the method used in deriving the Harvard photographic magnitudes:

> An absolute scale of magnitudes was derived separately from each of about 80 plates taken by the above methods, the starting point in every case

being the mean photometric magnitude of such stars in the Polar Sequence given in Table I, as were measured on that plate.

This statement seems to imply that the zero point of the photographic scale has been based upon photometric, that is, visual magnitudes. As long as the visual standards used are near the sixth magnitude, this means only the adoption of the international zero point. Nor for fainter stars would there be any difficulty in using visual standards were it certain that the visual and photographic scales for any given type of spectrum coincided. This, however, appears not to be the case. Adams,[1] for example, finds striking evidence that stars of the same spectral type may show a very different distribution of intensity in the continuous spectrum background. The color indices in such cases must be different, and the scales which express the corresponding visual and photographic magnitudes cannot coincide. Again, the curve for the minimum color index indicates that the scales do not coincide even for the whitest stars.

This being the case, it follows that visual standards other than those near the sixth magnitude will not give reliable results for the zero point of the photographic scale. If now, as the above quotation seems to indicate, visual standards were used whenever possible, the result would be a necessary coincidence of the visual and photographic scales, at least for the white stars. For an individual plate the divergence assumed to exist between the true scales might appear; but when the results for plates covering different intervals were combined the overlapping portions would cause the divergence to disappear from the mean and produce an apparent coincidence. The photographic zero points for the Mount Wilson plates, on the other hand, were found by a process equivalent to a direct comparison with stars near the sixth magnitude.[2] If the interpretation of the quotation from *H.C.*, No. 170 is correct, the two photographic scales could not agree unless the true photographic and visual scales for the white stars

[1] *Contributions from the Mount Wilson Solar Observatory*, No. 78; *Astrophysical Journal*, **39**, 89, 1914.

[2] *Contributions from the Mount Wilson Solar Observatory*, No. 70; *Astrophysical Journal*, **38**, 241, 1913.

coincide. The latter alternative, as already pointed out, seems to be excluded.

This explanation also accounts for the disappearance of the divergence between the Harvard and Mount Wilson photographic scales near the tenth magnitude. Beyond thirteen there is but one visual magnitude for a white star given in *H.C.*, No. 170, so that for the fainter stars photographic standards presumably were used for the determination of the zero point. Moreover, any divergence between the true visual and photographic scales would begin to show at a point some two magnitudes or so above this limit owing to the absence of a smoothing effect in the lower part of the interval covered by the plates on which the faintest visual standards were used.[1]

One further point requires mention. Reference has been made to a divergence between the two photographic scales beyond magnitude 15.5. The Mount Wilson results in this region are still uncertain, as they are based upon stars at the limit of visibility. Had the Harvard values been used, the upward trend of the bounding curve would have been even greater than that shown. The curve itself, however, is probably not greatly in error, for if the Mount Wilson photographic magnitudes are here systematically too bright, it is likely that a similar error affects the visual magnitudes. Plates of longer exposure will be required to settle the point.

[1] During the discussion following the reading of the paper the following statement by Miss Leavitt was presented by Professor Pickering:

"The magnitudes of stars in the Polar Sequence were originally reduced in accordance with the statement in *H.C.*, No. 170, as quoted by Professor Seares, the zero point for each plate being made to coincide with the photometric magnitudes of stars measured on that plate. In order to avoid any error involved in employing stars other than those near the sixth magnitude, a new reduction was made, which, however, gave magnitudes identical with those in *H.C.*, No. 170. For each of the 20 groups, first differences between the magnitudes of successive stars were taken. Adding together the means of these differences gave a scale of magnitudes which was the mean of the 20 individual scales, and independent of the photometric scale. These magnitudes were then reduced to a zero point depending on the photometric magnitudes of stars of Class A between the magnitudes 5.5 and 6.5. The resulting magnitudes, as has been stated, coincided with the magnitudes in *H.C.*, No. 170."

This apparently leaves the question of the divergence between the Harvard and Mount Wilson photographic scales for the brighter stars still unexplained.

If the systematic change of color with magnitude be regarded as established, it becomes at once a matter of interest to determine whether the change is the same for all parts of the sky. For a preliminary examination several regions have been photographed and partial results for one, the field of the variable *S Cygni*, are also given in Fig. 1. The 200 color indices shown are based upon photographic and visual magnitudes obtained by comparisons with the Pole. The mean of three plates was used for each scale, and the accordance between the separate comparisons is good throughout. Diaphragm plates on *S Cygni* for the determination of the magnitudes of the fainter stars were also made, but these have not yet been reduced.

It will be observed that the curve of minimum index is here the same as for the polar region, and it is of interest to note how sharply the limit is defined for the lower end of the scale where the stars are numerous. The sudden increase in the minimum index beyond magnitude 15.5 is probably apparent, and means only that stars with smaller indices are too faint visually to appear on the plates measured.

It is of interest to compare these results with Hertzsprung's measures of effective wave-length for *N.G.C.* 1647. As already stated, his minimum at magnitude 14.5 is λ 4320, corresponding for bright stars to a spectrum of F0 and a color index of $+0.4$ mag. This is the same as the limit given by the color index curves. Should it prove that the color change for *N.G.C.* 1647 is the same as that for the *S Cygni* and polar regions, the agreement would afford a valuable control of the photographic and visual scales.

The final interpretation of these results cannot now be given, but certain general conclusions may be indicated. Large color index may mean an advanced spectral type, or it may mean a peculiar distribution of intensity in the continuous spectrum similar to that found by Adams. Were we to exclude the latter possibility we should be confronted with a curious result. The faint stars, none of which show any negative, or even any small positive indices, may be faint either because of small luminosity or because of great distance. That stars of small luminosity should show advanced spectra is not surprising. In fact, for those cases in which the

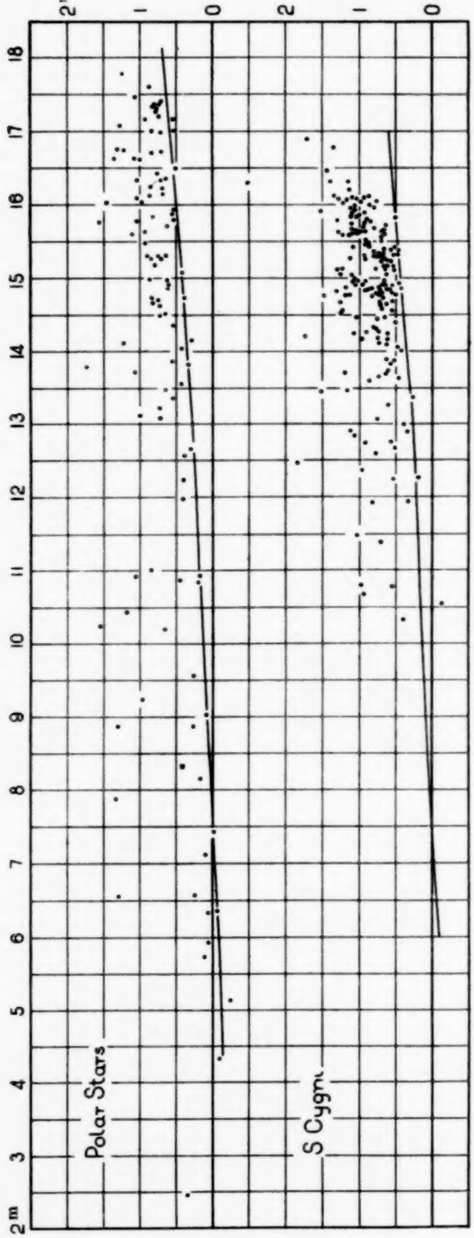

Fig. 1.—Variation of color index with photographic magnitude

absolute magnitudes are known, we find a rapid increase in spectral type with increasing absolute magnitude. But that stars which are faint because of great distance should show no early-type spectra would scarcely be expected, especially when the uniform change in minimum color index is considered. It would imply a gradually increasing suppression of the early types with increasing distance, and any such spectral distribution must be regarded as very improbable. Such is the conclusion resulting from the assumption that the observed increase in color index represents an actual change in spectral type.

The other alternative, namely, that the phenomenon is related to a change in the intensity distribution of the continuous spectrum, may be dependent upon either one or both of two factors—absolute luminosity and space absorption. Kapteyn has shown that, on the basis of the most reliable determinations of star density, the average luminosity must decrease with increasing apparent magnitude. Differences in luminosity may, however, mean differences in the general atmospheric absorption, even for stars of the same spectral type; and it is conceivable that the observed phenomenon might thus be accounted for. The presence of an absorbing medium in space, on the other hand, would also account for the gradual increase in the minimum color index. The results here presented do not permit a separation of the two effects. This phase of the question is, however, soon to be discussed exhaustively by Professor Kapteyn.

Mount Wilson Solar Observatory
December 19, 1913

THE SPECTRA OF MAGNESIUM, CALCIUM, AND SODIUM VAPORS

By JAMES BARNES

The study of the changes produced in the spectra of the elements by different forms of electrical discharges and by varying the pressure and chemical nature of the gas surrounding the electrodes has been the subject of a number of important investigations. The results so obtained have been found very useful in the interpretation of the physical conditions existing in sun-spots as well as in other fields of astrophysics. To these results the author wishes to add a few further observations.

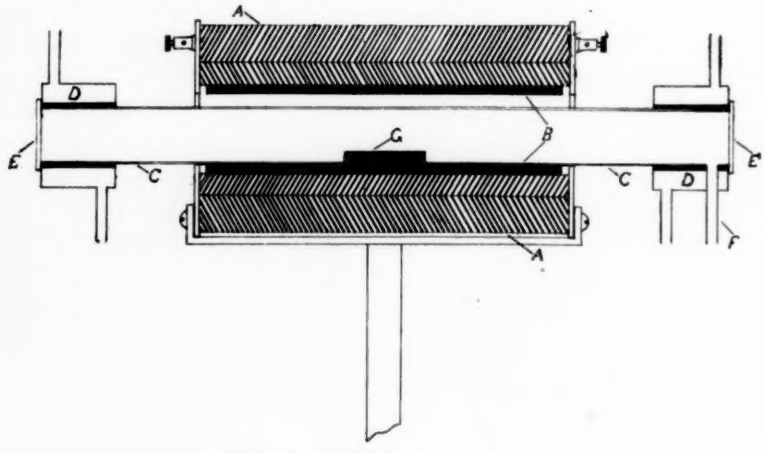

FIG. 1

The arrangement of the apparatus used is shown in Fig. 1. *A* is an electric furnace which was made by winding nichrome wire (No 17, B. and S.) about an alundum tube, *B*. This tube is 10 in. long and 2 in. in diameter and around it is placed a thick pipe-covering of 85 per cent magnesia which in turn is coated with asbestos cement. A long silica tube, *C*, 25 in. long and 1 in. in diameter was run through the alundum tube. In later experiments a glazed porcelain tube of about the same dimensions was used.

PLATE VI

Magnesium

—4481 —4571

Discharge in furnace

Air *in vacuo*

Arc in air

—6573

Calcium

Discharge in furnace

Arc in air

The ends of these tubes were closed by iron caps, *D*, and quartz plates, *E*. A stream of water running through the caps keeps them sufficiently cold so that the wax, by which they are sealed to the tube and to the quartz plates, does not melt. The tube *F* is connected with a Geryk exhaust pump.

A piece, *G*, of the metal under investigation is placed in the center of the tube, the caps adjusted, the air removed, and the furnace started. By regulating the current, the temperature of the furnace is held at such a point that the metal is slowly vaporized. The vapor is rendered luminous by the discharge from a large induction coil, parts of the caps, *D*, acting as internal electrodes.

The light coming from one end of the tube was analyzed by a Rowland concave grating of 6 ft. radius, and from the other end by a Hilger prism spectrometer of the constant deviation type. The intensity of the discharge was so strong for the substances used that good photographs of their spectra could be obtained in a few seconds with the prism instrument and in about ten minutes with the grating.

OBSERVATIONS

Magnesium.—I shall cite only the cases where the relative intensities of the lines in the spectrum of magnesium produced by this form of excitation are considerably different from that of the arc in air and *in vacuo* and from that of the spark. Reproductions from the negatives are shown on Plate VI. The strong spark line, λ 4481, which has been the subject for a large amount of investigation and discussion, is very weak in the furnace discharge, in fact on many plates it cannot be found. The strongest lines are λ 2852 and λ 4571. The remarkable intensity of λ 4571 is noteworthy. This radiation is relatively weak in the arcs in air and *in vacuo*.[1] Concerning the spectrum of magnesium, Adams[2] remarks, "The only magnesium line which shows any marked change is λ 4571.275, which is increased from 5 in the sun to 7 in the spot." Brooks[3] regards this radiation as characteristic of magnesium nitride. I tried the experiment of thoroughly washing

[1] *Astrophysical Journal*, **21**, 74, 1905.

[2] *Ibid.*, **30**, 105, 1909.

[3] *Ibid.*, **29**, 184, 1909.

out the tube with dry hydrogen and using magnesium which had been prepared by melting a piece of the metal and condensing it *in vacuo.* The spectrum was not changed, the radiation λ 4571 being just as strong as found above.

The three bands with heads at λ 5622, λ 5211, and λ 4845 are strongly depicted on the spectrum plates. These bands have been attributed to a magnesium and hydrogen compound and their wave-lengths were found by Fowler[1] to correspond with many lines in sun-spot spectra. Recently he found[2] and measured the wave-lengths of a number of additional triplets in the ultra-violet region of the spectrum of the magnesium arc *in vacuo.* These lines also occur in the furnace spectra and were easily found on plates taken with a Hilger quartz spectroscope.

Calcium.—The intensities of the H and K lines and λ 4227 as produced in the furnace discharge do not differ very much from their values as produced in the arcs in air or *in vacuo.* The only line which is strongly affected is λ 6573 as can be seen on Plate VI and is one of the strongest lines in the spectrum. This line has been found by Hale and Adams[3] to be "one of the most strongly affected lines in the entire spot spectrum, showing a rise of intensity from 1 in the sun to 10 in the spot." The bands with heads at λ 6382 and λ 6389 which are also an important feature of sun-spot spectra also occur in the furnace discharge but not with as great an intensity as was found in the arc *in vacuo.*

Sodium.—The spectrum of sodium vapor in the furnace shows the D lines with their intensities very much increased relatively to the other lines. The wings on the D lines are very wide and clear. Adams (*loc. cit.*) found this to be also the case in sun-spot spectra.

In the light of these observations, the author wishes to conclude that the physical conditions density, temperature, and excitation, as used above give spectra which are the nearest he has yet produced to those observed in sun-spots.

Bryn Mawr College
November 1913

[1] *Phil. Trans. Roy. Soc.*, **209** A, 447, 1909.

[2] *Proc. Roy Soc.*, **89** A, 137, 1913.

[3] *Astrophysical Journal*, **30**, 92, 1909.